The Incremental Rigidity Method – More-Direct Conversion of Strain to Internal Force in an Instrumented Static Loading Test

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ABSTRACT

To convert strain to internal force, it is conventionally assumed that both the foundation's composite-section elastic modulus, E, and total cross-sectional area, A, are known. However, only the product EA, the foundation's ***axial rigidity***, is needed. Using applied test loads and measured strains, the Incremental Rigidity ("IR") method determines the relationship between axial rigidity and strain at individual strain gage levels. From this relationship, measured strains can be converted to internal forces without having to know either a deep-foundation element's elastic modulus or its cross-sectional area. This paper reviews several published methods commonly used to estimate elastic modulus, cross-sectional area, internal stress, and internal force in a deep-foundation element. The assumptions, uncertainties, and limitations of each method are discussed. The IR method for converting strain to internal force in an instrumented static loading test is presented, including assumptions, uncertainties, limitations, and best practices associated with method.

Keywords: Incremental Rigidity, Axial Rigidity, Static Load Testing, Load Transfer, Internal Forces, Instrumentation, Strain, Strain Gages.

INTRODUCTION

Static loading tests have an important role in the design and construction of deep foundations of all types: driven piles, augered cast-in-place piles ("ACIP"), drilled displacement piles ("DDP"), helical piles, drilled shafts, etc. For simplicity, all deep foundation types will be referred to herein as piles. Additionally, the types of static loading tests addressed herein are axial, both head-down and bi-directional compression, and tension. The usefulness of static loading tests, particularly in the design phase, is enhanced by determining load-transfer response during the test. Load-transfer response refers to the manner in which internal pile forces are transferred into the surrounding geomaterial, with the definitive result being the magnitudes of mobilized unit shaft resistances along the pile length versus relative soil-pile movement. Integral to this objective is determining internal forces at various locations within the deep-foundation element.

Load-transfer measurements can be obtained by several different types of instrumentation (Brown et al. 2018); two common types are telltales and strain gages. Neither instrumentation type measures internal forces directly. When detailed load-transfer data is desired, telltale measurements alone are insufficient (Hannigan et al. 2016). Weldable strain gages can be used on steel pipe and H-piles, and "sister bar" strainmeters or concrete embedment strain gages can be installed in concrete(d) piles.

The reliability of strain measurements may be excellent, but conversion of strain measurements to internal pile force is not necessarily straightforward (Sinnreich 2011), as it is a function of the pile's physical characteristics at the strain measurement's location. This reality is at best misrepresented, and at worst misunderstood, by the notion that strain gages measure internal forces (rather than provide readings that are an intermediate step in calculating internal forces). The physical characteristics required to convert strain to internal force can vary by location within the pile. Some can be measured, but are more-often assumed, assigned presumptive values, estimated, based on constitutive relationships, or back-calculated. The methods used to determine a pile's physical characteristics can introduce significant error into internal pile force calculation, and therefore into determined mobilized unit shaft resistances. The Incremental Rigidity ("IR") method offers a more-direct determination of a pile's physical characteristics, and therefore a more-direct conversion of strain measurements to internal forces and mobilized unit shaft resistances.

FROM STRAIN TO INTERNAL FORCE

Static loading tests are typically performed using step-and-hold test load increments. For each load increment of a static load test, internal pile forces are calculated at each strain gage ("SG") level, and a resulting internal force profile (internal force as a function of depth or elevation) is determined. Internal pile forces at each SG level are calculated using the average measured strain at that SG level (if the SG level contains more than one strain gage), and the product of the pile's composite-section elastic modulus and cross-sectional area at that SG level by the following relationship:

$$F_i = E_i A_i \varepsilon_i \quad \text{[Eq. 1]}$$

Where at each SG Level i: F_i = Internal pile force
E_i = Pile composite-section elastic modulus
A_i = Pile cross-sectional area
ε_i = Pile strain

Since during the static loading test strain is the measured parameter in Eq. 1, conversion of strain to internal force has conventionally involved somehow determining the cross-sectional area and composite-section elastic modulus at each strain gage level.

Cross-Sectional Area

For driven piles, such as steel pipe, H-piles, and prestressed concrete piles, the foundation cross-sectional areas at strain gage levels are generally accurately known. For drilled foundations, the cross-sectional areas at SG levels can be based on a permanent casing diameter, mechanical or sonic profiling, thermal profiling, drilling tool dimensions, or incremental concrete volume placement measurements. With the exception of permanent casing diameter, all these methods of assessing cross-sectional area contain varying degrees of uncertainty in their determinations. For composite-section piles (most-often comprised of steel and concrete), in addition to the total cross-sectional area, the relative areas of steel and concrete must also be known. For simplicity, "concrete" will be used herein to refer to both concrete and grout. The area of steel is generally accurately known; any uncertainty in cross-sectional area determination usually translates into uncertainty about the area of concrete.

Composite-Section Elastic Modulus

Instrumented piles subject to static loading tests are generally comprised entirely of steel, entirely of concrete, or a combination of these two materials (composite-section piles). The elastic modulus of steel is generally taken to be 29.0 to 30.0×10^6 pounds per square inch ("psi"). To determine the elastic modulus of a pile comprised entirely of concrete, or of a composite-section pile, the concrete's elastic modulus, E_{CONC}, must be determined.

A comprehensive comparison of methods to determine the elastic modulus of concrete and composite-section piles was performed by Lam and Jefferis (2011). Since steel's elastic modulus is known, the methods presented focus on determining the concrete modulus. Ten methods were identified: four based on laboratory tests, and six on in-situ pile instrumentation. Two of the more-popular of these methods, one based on laboratory tests and one based on in-situ pile instrumentation, are discussed below.

<u>ACI Relationship</u> – A number of empirical relationships exist to estimate the elastic modulus of concrete based on unconfined compression strength determined from test cylinders as reported in the ACI Committee Report 363-10 (2010). One of the more-popular relationships is offered by the American Concrete Institute ("ACI"). According to the ACI 318-14 manual (2014), the relationship between concrete elastic modulus and unconfined compressive strength can be given by the following:

$$E_{CONC} = w_c^{1.5} 33(f'_c)^{0.5} \text{ (psi)}; \quad E_{CONC} = w_c^{1.5} 0.043(f'_c)^{0.5} \text{ (MPa)} \qquad \text{[Eq. 2]}$$

$$\text{for } 90 \text{ pcf} < w_c < 160 \text{ pcf}; \quad 14 \text{ kN/m}^3 < w_c < 25 \text{ kN/m}^3$$

Where: E_{CONC} = Concrete elastic modulus
w_c = Concrete unit weight
f'_c = Concrete test cylinder unconfined compressive strength

For normal-weight concrete, ACI permits the concrete elastic modulus to be given by the following relationship:

$$E_{CONC} = 57{,}000(f'_c)^{0.5} \text{ (psi)}; \quad E_{CONC} = 4{,}700(f'_c)^{0.5} \text{ (MPa)} \qquad \text{[Eq. 3]}$$

Eqs. 2 and 3 represent results of the work completed and presented by Pauw (1960) where the relationship between concrete unconfined compressive strength and unit weight was explored. When these two expressions were developed, average concrete strengths were significantly lower, potentially on the order of half the strength, of those encountered currently in practice. This discrepancy has led scholars and industry experts to perform further research, and to develop new correlations (Ahmad & Shah 1985, Smith et al. 1964, Freedman 1971, Burg & Ost 1994, Iravani 1996, Mokhtarzadeh & French 2000 a & b). Although standard practice continues to follow the expression shown in Eqs. 2 and 3, synthesized descriptions of the above-mentioned research work, along with newer correlations, are published in ACI Committee Report 363R-10 (2010) (Fig. 1).

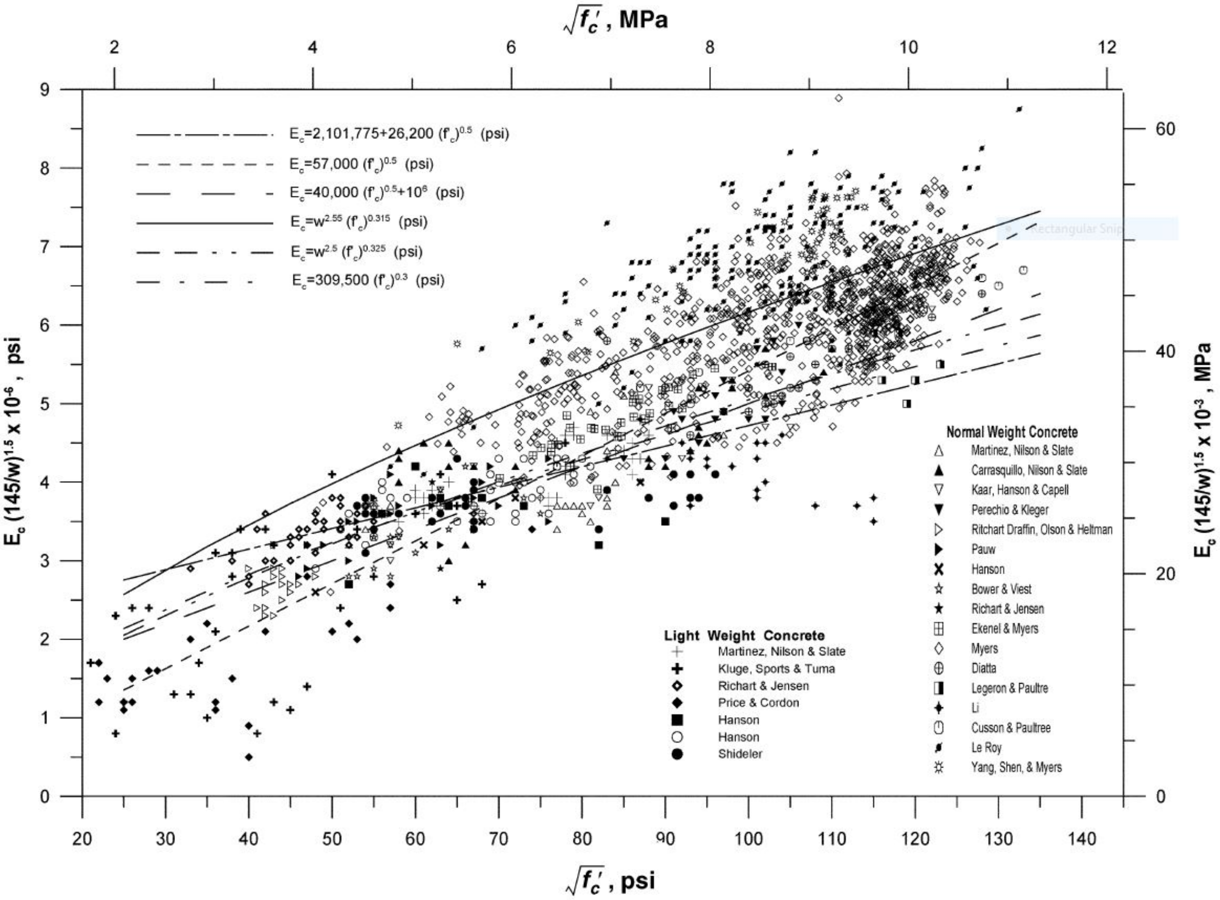

Fig. 1. Modulus of elasticity versus square root of concrete strength, incorporating lower- and higher-strength concrete data (adapted from Myers and Yang (2004)

The following are offered regarding the uncertainty in E_{CONC} assessment using the ACI relationship:

1. Hayes & Simmonds (2002) conclude that Eq. 2 may not always provide reasonable values for E_{CONC} when converting strains to internal forces.
2. Eq. 2 is based on regression through scattered data, and is just one of many potential formulas among which to choose (Fig. 1).
3. Eq. 2 is based on the definition of E_{CONC} as the slope of the line drawn from a stress of zero to a compressive stress of $0.45f'_c$. E_{CONC} may vary, and therefore the uncertainty associated with Eq. 2 may increase, at stress levels other than $0.45f'_c$.
4. The value of E_{CONC} is more-dependent on the unit weight of concrete, and the test method used to determine it, than on the concrete compressive strength (Pauw 1960). Eq. 2 implies that for an increase in concrete unit weight from 140 to 155 pounds per cubic foot ("pcf"), E_{CONC} increases by over 16%.
5. Other researchers (Ahmad & Shah 1985, Freedman 1971) have concluded, and ACI 318-107 reports, that Eq. 2 predicts E_{CONC} within ±20%.
6. E_{CONC} is sensitive to the elastic modulus of the aggregate (ACI 318-14, 2014). This is not accounted for in Eq. 2.

7. f'_c determined from concrete cylinders in the laboratory may not reflect the strength of the mass concrete in the pile. Hayes & Simmonds (2002) state that it is generally acknowledged that E_{CONC} correlates well to the 0.5 power of the compressive strength of the mass concrete in the pile. Studies by Khan et al. (1995) suggest that normal test cylinder curing methods result in values of laboratory-determined f'_c that are significantly lower than the compressive strength of the mass concrete in the pile.
8. Komurka and Robertson (2020) offer evidence that E_{CONC} may be a function of depth/elevation within a pile, even for piles with constant cross-sections such as concrete-filled pipe piles (perhaps the fluid pressure to which concrete is subjected during curing affects the concrete's final unit weight and modulus).
9. E_{CONC} is not constant over the strain range imposed by most static loading tests as implied by Eq. 2, but decreases with increasing strain (Fellenius 1989). Komurka and Robertson (2020) demonstrated that E_{CONC} can decrease by as much as 36% over the strain range imposed during a static loading test.
10. Due to lack of similar research on the relationship between grout strengths determined from test cubes and grout modulus, the ACI relationship developed from concrete cylinder testing is often applied to grout cube results.

<u>Tangent Modulus Method</u> – The Tangent Modulus ("TM") method (Fellenius 1989, 2001, and 2019; Salem and Fellenius, 2017) is a commonly-used method to estimate E_{CONC} that offers advantages over using the ACI relationship. It is based on the premise that after a pile's shaft resistance is fully mobilized between a test load location and a strain gage level, subsequent incremental increases in stress at the test load location result in proportional incremental increases in stress at the strain gage level. The quotient of change in stress divided by change in strain ($\Delta\sigma/\Delta\varepsilon$, the tangent modulus) plotted against strain resolves into a virtually straight line, sloping from a larger tangent modulus to a smaller one with increasing strain (Fig. 2). The line's negative slope indicates that E_{CONC} decreases with increasing strain.

The mathematics of the method are presented in Fellenius 2001and 2019, and are repeated here to allow for their comparison to the mathematics of the subsequently discussed Incremental Rigidity method. The equation for the tangent modulus line (Fig. 2) is:

$$\text{Tangent Modulus of Composite Pile Material} = (d\sigma/d\varepsilon) = a\varepsilon + b \qquad \text{[Eq. 4]}$$

Which can be integrated to:

$$\text{Stress in the Pile, } \sigma = 0.5a\varepsilon^2 + b\varepsilon \text{ (the integration constant can be assumed equal to 0)} \qquad \text{[Eq. 5]}$$

An alternative approach to determine the stress in the pile, σ, is:

$$\sigma = E_{SEC}\,\varepsilon \qquad \text{[Eq. 6]}$$

Therefore, the secant modulus, E_{SEC}, is determined as:

$$E_{SEC} = 0.5a\varepsilon + b \qquad \text{[Eq. 7]}$$

Where: σ = stress (load divided by cross-sectional area)
$d\sigma$ =change in pile head stress from one load increment to the next
ε = measured strain at a strain gage level
$d\varepsilon$ = change in measured strain at a strain gage level from one load increment to the next
a = tangent modulus line slope
b = tangent modulus line y-intercept (i.e., initial tangent modulus at zero strain)
E_{SEC} = composite-section secant modulus

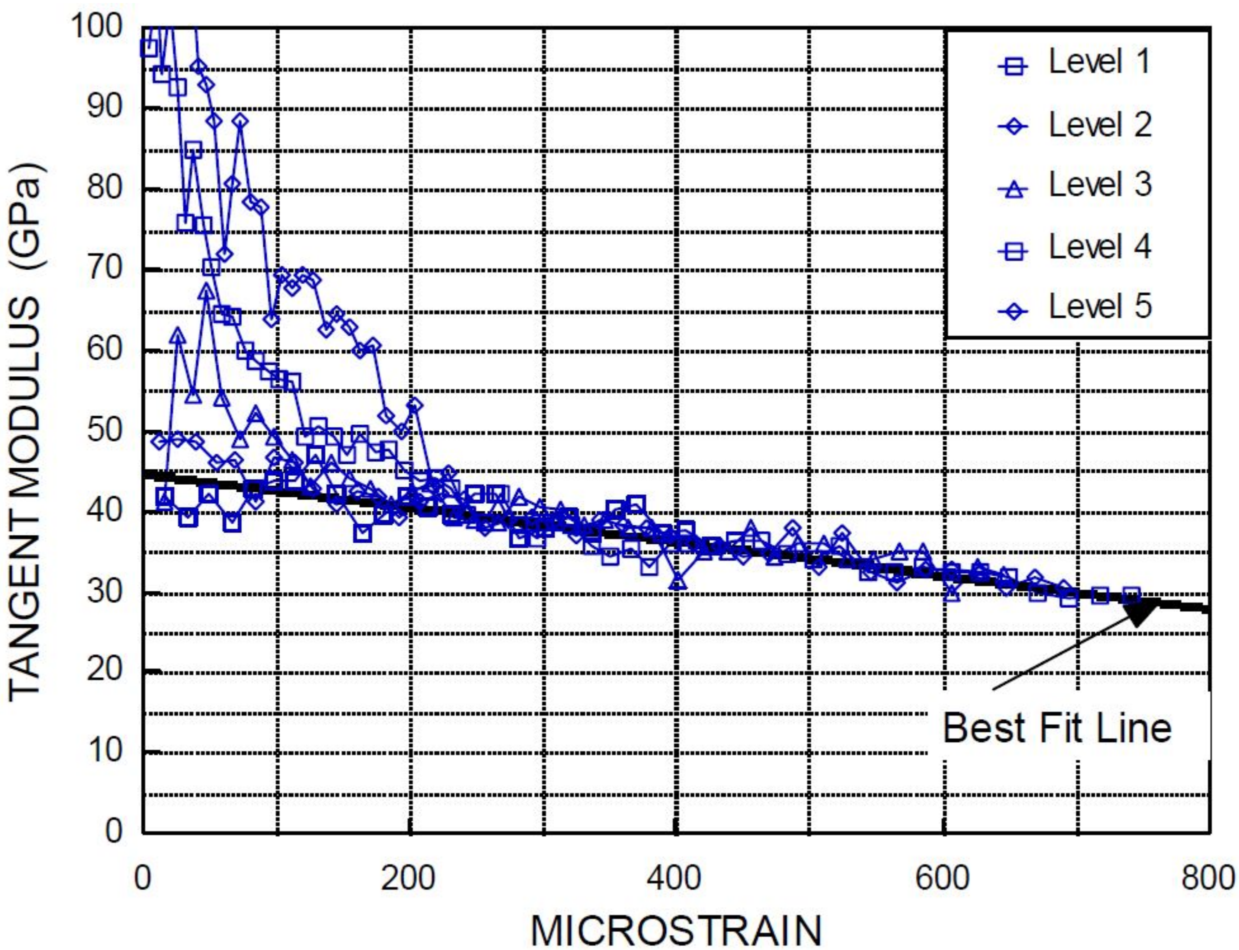

Fig. 2. Tangent modulus diagram (after Fellenius 2001)

A best-fit line (Fig. 2) is determined for the data from all strain levels, ignoring the initial, non-linear values caused by incompletely mobilized shaft resistance. By determining a single composite-section secant modulus relationship to strain, measured strain values can be converted to internal pile stress at all strain gage levels. The internal force in the pile at the strain gage level is then obtained be multiplying the internal pile stress by the pile cross-sectional area.

The Tangent Modulus method offers advantages over the ACI relationship in that it addresses many of the shortcomings of the ACI relationship. Two of the most-significant shortcomings addressed are that it is based on the response of the mass concrete in the pile (as opposed to a correlation to lab results), and it accounts for the strain-dependency of E_{CONC} (as opposed to assigning a constant value at all strains). Meaningful application of the TM method requires that its basic assumptions are met, namely 1) all the shaft resistance is fully mobilized between a test load location and a strain gage level which contributes to determining a linear composite-section secant modulus relationship to strain, 2) the cross-sectional area (both the total, and the respective areas of steel and concrete) are the same at the test load location and at the strain gage level, 3) E_{CONC} is constant at all strain gage levels, and 4) the soils between a test load location and a strain gage level exhibit no strain-hardening or strain-softening response. Test load locations include the pile head for a head-down loading test, and the jack assembly's upper and lower bearings for a bi-directional test.

Incremental Rigidity Method

Strain gages are installed along the length of a static loading test pile to aid in determining internal force profiles for each applied test load. Eq. 1 indicates that to convert measured strains to calculated internal forces involves the pile composite-section elastic modulus at the strain gage level, E, and the pile cross-sectional area at the strain gage level, A. The modulus is an intensive quantity (i.e., a physical quantity whose magnitude is independent of the size of the system (Child 1917, & Mills 1993)). Commonly, values of E and A are determined separately. However, knowledge of these two individual values is not strictly required in this application; only knowledge of their product, EA, is required.

In the literature, the product EA has been referred to as "stiffness"; this is a misnomer. Stiffness is EA/L (Baumgart 2000, Goodno & Gere 2017, Holscher & van Tol 2008), and so has units of force divided by length. The product EA is ***axial rigidity*** (Goodno & Gere 2017, Greenspan 1943 & 1946, Pelecanos et al. 2017, Vable 2008), and so has units of force. For simplicity, it will be referred to herein as rigidity. Rigidity is an extensive quantity (i.e., a physical quantity whose magnitude depends on the size of the system (Child 1917, Mills 1993)). That is, rigidity is a property of a solid body that depends on its constitutive material(s), and its shape and boundary conditions (i.e., specific to a location within a test pile). As such, it is precisely what is required to convert strain to internal force at a particular strain gage level.

The Incremental Rigidity method is based on the Tangent Modulus method, but instead of relating changes in stress to changes in strain to determine a modulus relationship, the IR method relates changes in applied test load to strain to determine a force relationship. In this way, IR provides a more-direct conversion of measured strain to internal force. Similar to the TM method, the IR method is based on the premise that after a pile's shaft resistance is fully mobilized above a strain gage level, subsequent incremental increases in test load at the pile head result in proportional incremental internal force increases at the strain gage level. The quotient of change in test load and change in strain ($\Delta Q/\Delta\varepsilon$, incremental rigidity) plotted against strain for an individual strain gage level resolves into a virtually straight line, sloping from a larger rigidity to a smaller one with increasing strain (Fig. 3). A comparison of Figs. 2 and 3 shows that the vertical axis of an incremental rigidity plot differs from the vertical axis of a tangent modulus plot only by dividing by the pile cross-sectional area (the difference between load and stress).

Rearranging Eq. 1 and using load applied to the pile head results in:

$$Q/\varepsilon = EA = \text{Rigidity of Composite Pile Material} \qquad \text{[Eq. 8]}$$

Working with changes in load applied to the pile head and strain, the equation for the incremental rigidity line (Fig. 3) is:

$$\text{Incremental Rigidity of Composite Pile Material} = (dQ/d\varepsilon) = a\varepsilon + b \qquad \text{[Eq. 9]}$$

Which can be integrated to:

$$\text{Internal Force in the Pile} = 0.5a\varepsilon^2 + b\varepsilon \text{ (assuming the integration constant = 0)} \qquad \text{[Eq. 10]}$$

Where: Q = load applied to the pile head
dQ = change in load applied to the pile head
ε = measured strain at a strain gage level
$d\varepsilon$ = change in measured strain at a strain gage level from one load increment to the next
a = incremental rigidity line slope
b = incremental rigidity line y-intercept (i.e., initial rigidity at zero strain)

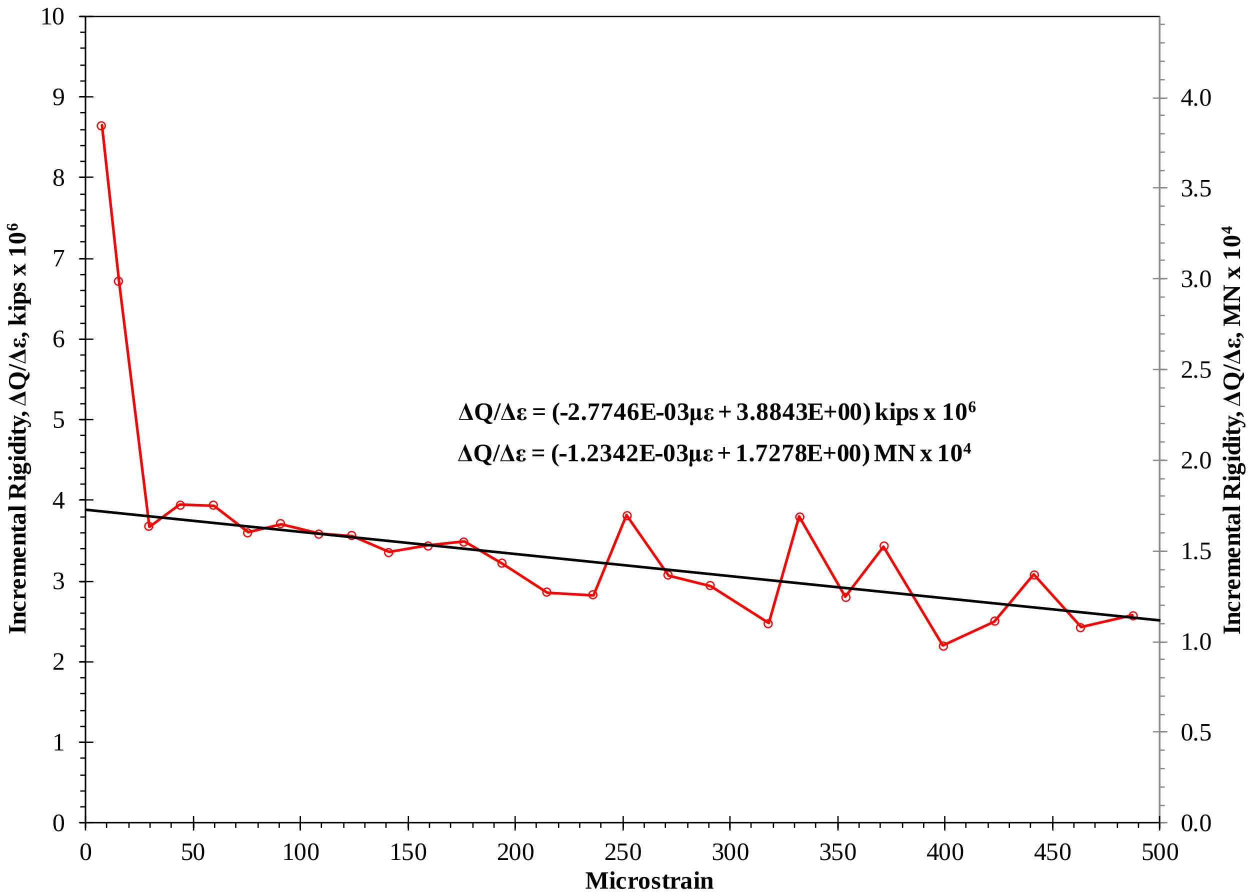

Fig. 3. Incremental Rigidity Diagram for One Strain Gage Level

Inspection of Eqs. 8 and 9 indicates that using the Incremental Rigidity method, internal forces in the pile can be determined without knowledge of separate values for modulus or area. Accordingly, the IR method offers all the advantages of the Tangent Modulus method, with the added benefit that it can be applied to strain gage levels within non-uniform piles. In this context, non-uniform refers to piles whose total cross-sectional area, and/or the respective areas of steel and concrete, and/or E_{CONC}, varies by location along the pile length.

Even for apparently uniform piles, it is recommended that IR results be evaluated, and potentially different relationships be used, for individual strain gage levels. It has been demonstrated that even in apparently uniform piles, concrete modulus (both its initial (zero-strain) value, and its strain-dependent relationship) can vary by location along the pile length (Hong et al. 2019, Komurka and Robertson 2020).

Some judgment must be applied in deciding which strain gage levels provide interpretable results, and which do not. The Incremental Rigidity method provides more-interpretable results for strain gage levels closer to the test load than for SG levels farther from the test load (Komurka and Robertson 2020). This is because at SG levels closer to the test load, shaft resistance between the test load and the SG level is more-likely to be fully mobilized. The method also provides less-interpretable results for SG levels near the ends of a static loading test pile (i.e., in a head-down compression or tension test, near the base/toe; in a bi-directional test, near the head and near the base/toe). This is because if the pile end displaces or creeps during the test, strains in the pile near the displacing end relax and decrease, and the ratio of test load to measured strain is not linear with respect to strain but instead increases. This manifests itself as an increasingly positive slope in an IR plot (Komurka and Robertson 2020). Fellenius and Ochoa 2009 offer additional explanation of how strain-softening response can affect results.

It is recommended that, whenever possible, strain be accurately determined at instrumentation levels for conversion to internal force. However, if incorrect gage factors are inadvertently applied, or if gage factors are unknown, the Incremental Rigidity method can still be applied to calculate internal forces at individual interpretable strain gage levels. Similarly, the IR method can be applied to measurements that report alternative units (e.g., digits for vibrating-wire strain gages, volts for resistance-type strain gages) whose relationship is linear with respect to changing strain.

Implementing a number of static loading test and data reduction protocols aids in improving Incremental Rigidity method analyses (it is assumed that since numerous strain gage readings are being obtained from multiple strain gage levels, a datalogger is being used):

1. Strain gages should be located a minimum of two pile diameters from the load application, and one diameter from the pile toe/base. For sister bar strainmeters, this dimension ideally refers to the end of the sister bar, not to the centerline of the strainmeter.
2. Consider an increased frequency of strain gage levels near the test load.
3. Avoid unload/reload cycles (Fellenius 2019, Siegel 2010). Cycling the load induces unrecoverable internal compressive load that varies non-uniformly along the pile length; shaft resistance distribution and direction will vary with depth and make strain gage data interpretation difficult (Siegel 2010).
4. Apply equal load increments, target a minimum of 20 increments.
5. Hold each applied load as constant as possible. If the test is controlled by monitoring jack pressure, use a digital pressure gage.
6. Use equal hold times for each applied load increment.
7. Longer applied-load hold times are better than shorter ones; suggest 8 to 10 minutes minimum. Longer hold times provide more time for internal forces to “migrate” to the lower portions of a test pile, and for strains to stabilize (especially important for long

friction piles), and they provide more pressure and strain data points to average for a given applied load increment.

8. Record jack pressures using a pressure transducer, and load cell readings (if used), using the same datalogger that is reading the strain gages, so that pressure/load and strain readings occur at the same time and can be accurately related in time to each other.
9. Use relatively short datalogger reading intervals, on the order of 30 seconds or less.
10. When selecting values of applied load and measured strain for a given load increment, as opposed to selecting a single value from the last recorded row of datalogger data, use the average of representative load and strain readings over the hold time. Using average values can make the Incremental Rigidity plots more-interpretable (Komurka and Robertson, 2020).

CONCLUSIONS

Axial static loading tests are valuable tools for the deep-foundation industry. Static loading test results are enhanced by determining load-transfer response during the test. To this end, strain gages are commonly installed along the length of a static loading test pile to aid in determining internal force profiles. Conversion of strain to internal force is not necessarily straightforward, and has conventionally involved somehow determining or estimating the cross-sectional area and composite-section elastic modulus at each strain gage level.

Many methods exist for assessing cross-sectional area; the methods contain varying degrees of uncertainty in their determinations. Since the modulus of steel is fairly well-known, estimation of composite-section elastic modulus generally focuses on determining the elastic modulus of concrete, E_{CONC}. Methods to determine E_{CONC} can be classified as either based on laboratory tests, or based on in-situ pile instrumentation. Two of the more-popular of these methods are the ACI relationship (based on laboratory tests), and the Tangent Modulus method (based on in-situ pile instrumentation).

The ACI relationship estimates E_{CONC} based on unconfined compressive strengths of concrete cylinders, and owing to a number of uncertainties, may not always provide reasonable values for E_{CONC} when converting strains to internal forces. The Tangent Modulus method offers advantages over the ACI relationship by determining the composite-section tangent modulus of uniform piles.

The Incremental Rigidity method offers further advantages. Rather than utilizing changes in applied stress, the IR method utilizes changes in applied load. Instead of requiring separate determinations of composite-section modulus, E, and total cross-sectional area, A, the IR method determines the product of these two parameters, EA (axial rigidity), to offer a more-direct conversion from strain to internal force. The IR method permits rigidity to be determined at individual strain gage locations along the pile length, allowing for internal force calculation in non-uniform piles.

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