Results and Lessons Learned from Converting Strain to Internal Force in Instrumented Static Loading Tests Using the Incremental Rigidity Method

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ABSTRACT

A valuable objective of performing static loading tests on deep foundations is to determine load-transfer response. Integral to this objective is determining internal forces at various locations within the deep-foundation element. These internal forces are usually obtained from strain measurements. Using applied test loads and measured strains, the Incremental Rigidity (“IR”) method determines the relationship between axial rigidity, $EA$, and strain at each strain gage level. This paper presents results from a number of instrumented static loading tests where the IR method was used to convert measured strains to internal forces. Various illustrations, conclusions, and lessons learned from application of the IR method to these case histories are presented. Topics presented include the strain dependence of rigidity, interpretable versus non-interpretable IR results, the strain magnitudes required to yield interpretable IR results, the use of at- or above-grade strain gages, the benefit of averaging test load and strain values, determining variable rigidities within an apparently uniform pile, determining rigidities at non-interpretable strain gage levels, and applying the IR method to complex geometries. Results from a closed-end concrete-filled steel pipe pile are presented which indicate that the elastic modulus of concrete may increase linearly with depth in the pile.

Keywords: Incremental Rigidity, Axial Rigidity, Static Load Test, Load Transfer, Internal Forces, Instrumentation, Strain, Strain Gages, Pipe Piles, Drilled Shafts, ACIP Piles, Bored Piles.

INTRODUCTION

Static loading tests have an important role in the design and construction of deep foundations of all types. For simplicity, all deep foundation types will be referred to herein as piles. The usefulness of static loading tests, particularly in the design phase, is enhanced by determining load-transfer response during the test, with the definitive result being determining the locations and magnitudes of mobilized unit shaft resistances along the pile versus relative soil-pile movement. Load-transfer measurements are commonly obtained by the use of strain gages. Weldable strain gages can be used on steel piles, and “sister bar” strainmeters or concrete embedment strain gages can be installed in concrete(d) piles.

Internal pile forces at each strain gage (“SG”) level are calculated using the average measured strain at that SG level (if the SG level contains more than one strain gage), and the product of the pile’s composite-section elastic modulus and cross-sectional area at that SG level by the following relationship:

$$F_i = E_iA_i\varepsilon_i \quad [\text{Eq. 1}]$$
Where at each SG Level \( i \): 
\[ F_i = \text{Internal pile force} \]
\[ E_i = \text{Pile composite-section elastic modulus} \]
\[ A_i = \text{Pile cross-sectional area} \]
\[ \varepsilon_i = \text{Pile strain} \]

Since during the static loading test strain is the measured parameter in Eq. 1, conversion of strain to internal force has conventionally involved somehow determining the cross-sectional area and composite-section elastic modulus at each strain gage level. Komurka and Moghaddam (2020) presented the Incremental Rigidity ("IR") method, which determines the product \( EA \), the foundation’s **axial rigidity** at a strain gage level. For simplicity, \( EA \) will be referred to herein as rigidity. The Incremental Rigidity method is based on the Tangent Modulus method (Fellenius 1989, 2001, and 2019; Salem and Fellenius, 2017), but instead of relating changes in stress to changes in strain to determine a modulus relationship, the IR method relates changes in applied test load to strain to determine a force relationship.

The quotient of change in test load divided by change in strain \( (\Delta Q/\Delta \varepsilon, \text{incremental rigidity}) \) plotted against strain for an individual strain gage level resolves into a virtually straight line, sloping from a larger rigidity to a smaller one with increasing strain (Komurka and Moghaddam 2020) (Fig. 1).

![Fig. 1. Incremental rigidity plot for one strain gage level depicting significant decrease over strain range induced during static loading test.](image-url)
The coefficients of the best-fit line from an incremental rigidity plot are used to determine internal forces at a strain gage level according to the following relationship (Komurka and Moghaddam 2020):

\[
\text{Internal Force in the Foundation} = 0.5a\varepsilon^2 + bc
\]  
[Eq. 2]

In this way, the Incremental Rigidity method offers a more-direct conversion of strain to internal force. Through the use of case history examples, this paper presents various conclusions drawn and lessons learned from application of the IR method. Since the vertical axis of an incremental rigidity plot differs from the vertical axis of a tangent modulus plot only by dividing by the pile cross-sectional area (Komurka and Moghaddam 2020), the conclusions and lessons presented herein may be considered to also be applicable to the Tangent Modulus method (Fellenius 1989, 2001, and 2019; Salem and Fellenius, 2017).

**INCREMENTAL RIGIDITY STRAIN DEPENDENCE**

Since incremental rigidity (EA) decreases with increasing strain, and the area can be considered virtually constant (the Poisson effect can be ignored), it follows that composite-section modulus must be decreasing with increasing strain. Since the modulus of steel is virtually non-strain-dependent (prior to yield), it follows that \(E_{\text{CONC}}\) or \(E_{\text{GROUT}}\) must be decreasing with increasing strain. A foundation’s decrease in rigidity over the strain range induced by a static load may occasionally be significant. For a given range of internal forces, the potential decrease in EA values is greater in slender foundations with lower rigidities which experience more strain than it is in stouter foundations with higher rigidities which experience less strain.

Fig. 1 presents an incremental rigidity plot for a strain gage level in a 24-inch-diameter grouted augered cast-in-place pile from a bi-directional static loading test (i.e., test loads applied by an embedded jack assembly). The depicted SG level was 4.3 feet below the jack assembly’s lower bearing plate in a bi-directional static loading test. The average of three grout cylinder uniaxial compressive strength test results performed on the day of the bi-directional loading test was 8,032 pounds per square inch (“psi”). Inspection of Fig. 1 indicates that incremental rigidity at the subject SG level decreased 36 percent over the strain range experienced during the test. Owing to the 0.5 factor in Eq. 2, this translates into an error of 18 percent in the calculated change of internal forces (and therefore unit shaft resistance values) from start to end of the test if a single, non-strain-dependent rigidity (i.e., \(E_{\text{GROUT}}\)) value would be used to convert measured strains to internal forces.

**REQUIRED STRAIN MAGNITUDE**

As shaft resistance between a test load application location and a strain gage level is increasingly mobilized, the strain gage level experiences increasing strain. After a foundation’s shaft resistance is fully mobilized between the test load and a strain gage level, assuming plastic response, subsequent incremental increases in test loads result in proportional incremental increases in strain at the SG level. When incremental increases become proportional to one another, the relationship between incremental rigidity and strain exhibits linear response. The magnitude of strain at which this linear response is exhibited depends on the foundation’s rigidity (Eq. 1).
For both compression and tension tests, the maximum internal force which can be developed at a strain gage level is equal to the sum of the static resistance “downstream” (beyond the strain gage on the other side) of the test load. For a given internal force, a foundation with a higher rigidity will exhibit less strain than a foundation with a lower rigidity. The incremental rigidity plot for a strain gage level located relatively near the test load, in a foundation with a large rigidity value relative to the internal forces produced during a static loading test, may resolve into linear response at relatively small strains.

Fig. 2 presents an incremental rigidity plot from a bi-directional static loading test (“BDSL”) performed on a 36-inch-diameter drilled shaft. The strain gage level presented was located 3.5 feet from the jack assembly’s lower bearing plate. The soil surrounding the shaft between the lower bearing plate (i.e., the test load) and the SG level consisted of soft clay. Applied load increments were approximately 20 kips. The shaft resistance in this foundation segment was mobilized fairly early in the test, as evidenced by the relatively small number of load increments applied (three) before the incremental rigidity plot exhibits linear response. This, in conjunction with the foundation’s rigidity, accounts for the IR plot exhibiting linear response at relatively small strains resulting from the internal forces achieved during the test after mobilization of the shaft resistance between the test load location and the SG level.

![36-Inch Drilled Shaft](image)

- Embedded Length = 30.0 feet
- Ground Surface Elevation = 123.5 feet
- Jack Assembly Elevation = 100.0 feet
- SG Level Elevation = 96.5 feet

$\frac{\Delta Q}{\Delta \varepsilon} = -4.4105 \times 10^{-2} \mu e + 6.8245 \times 10^0$

**Fig. 2.** Incremental rigidity plot for one strain gage level exhibiting linear response at relatively low strain magnitudes.
USE OF ABOVE- OR AT-GRADE STRAIN GAGES

Both head-down compression and tension static loading tests offer the opportunity to install at- or above-grade (or just below grade) external and/or internal strain gages (hereafter collectively referred to as top gages). The intended benefit is to determine pile properties (either composite-section tangent modulus, secant modulus, or rigidity) before any load transfer into the surrounding material occurs. However, care should be exercised in applying these results to locations lower in the pile, as they may be affected by a number of factors unique at or near the pile head, and therefore misrepresent conditions at lower locations. It is not uncommon for top strain gage levels to exhibit significantly different incremental rigidity relationships than lower SG levels.

Different curing conditions exist at the pile head than exist lower in the foundation. These different curing conditions may affect the concrete or grout modulus, and therefore the pile’s rigidity. Differences in curing temperature (both the magnitude, and variation, during curing) may exist between near-surface and lower locations. The pressure head to which the fluid concrete and grout is subject during curing may affect long-term modulus values. Piles may receive concrete from multiple trucks and different batches. Depending on the distance between the test load and the SG level, there may be stress concentration effects.

At the top strain gages’ location, there may be bending effects during the test, especially with slightly off-center internal gages, whereas at depth bending effects may be manifested differently, much-diminished, or absent. In the case of concreted-in-place pipe piles, debonding may occur between the pile’s steel shell and its internal concrete fill. This is apparent on a diagram of internal and external measured strain (at the same elevation) versus applied test load by the two plots potentially tracking nearly identically in the early portion of the test, then distinctly diverging for the remainder of the test (Fig. 3).

Fig. 3. Illustration of concrete fill and steel shell de-bonding during a head-down static loading test in a concreted-in-place pipe pile.
INTERPRETABLE VERSUS NON-INTERPRETABLE RESULTS

Some judgment must be applied in deciding which strain gage levels provide interpretable results, and which do not. Strain-dependent response exhibited in an incremental rigidity plot can vary significantly. As discussed by Komurka and Moghaddam (2020), the IR method can only be applied when the IR plot resolves to linear. However, even when linearity is obtained for an individual SG level, additional judgment must be exercised as to the applicability of this linear relationship in reducing SG measurements. Interpretable results are herein described as results from the IR plot which can be used for SG data reduction.

The Incremental Rigidity method provides more -interpretable results for SG levels closer to the test load than for SG levels farther from the test load. This is because at SG levels closer to the test load, shaft resistance between the test load and the SG level is more-likely to be fully mobilized. The method also provides less-interpretable results for strain gage levels near the ends of a static loading test pile (i.e., in a head-down compression or tension test, near the base/toe; in a bi-directional test, near the head and near the base/toe). This is because as the pile end displaces or creeps during the test, strains in the pile near the displacing pile end are relaxed and reduced, and the ratio of change in test load to change in measured strain is not linear with respect to strain but instead increases. This is manifest as an IR plot leveling out (becoming horizontal), or exhibiting an increasing positive slope (“J-hooking”). The distance away from a pile end at which an IR plot levels out or J-hooks depends on the ability of the foundation portion between the SG level and the pile end to solidly resist internal forces. Accordingly, this phenomenon can be especially pronounced near the base/toe of a tension test pile, as the base/toe contributes no resistance “downstream” of the SG level.

An example of this phenomenon is presented in Fig. 4. Fig. 4 presents the incremental rigidity plots from a 17.75-inch-diameter controlled-modulus column (“CMC”). Inspection of Fig. 4 indicates that Strain Gage Levels 1 and 2 (nearest the test load application) give interpretable results, exhibiting linear response similar to each other. The lower SG levels all appear trending toward exhibiting comparable response, but their trends are altered by leveling out, or by J-hooking (SG Level 6). Inspection of the SG level nearest the CMC base, SG Level 6, clearly indicates that the last seven strain readings were affected by base displacement, and the IR plot is therefore uninterpretable regarding a linear IR relationship. This aids in confirming that the leveling out of the last seven points in the IR plots of SG Levels 4 and 5 were also caused by base displacement, as opposed to being potentially interpretable linear response, making them also uninterpretable. Occasionally, these types of assessments can be corroborated by a break in the foundation head load versus displacement curve coinciding with the breaks in the trends of the affected IR plots, or by base telltale measurements. Conversion of strain to internal force at non-interpretable SG levels using IR results in a non-uniform pile is addressed in a subsequent section.

AVERAGING APPLIED TEST LOAD AND STRAIN VALUES

It is assumed that since numerous strain gage readings are being obtained from multiple strain gage levels, a datalogger is being used to record the readings. Jack pressures (using a pressure transducer) and load cell readings (if used) should be recorded using the same datalogger that is reading the strain gages, so that jack pressure/test load and strain readings can be accurately
correlated to each other by reference to a common time stamp. Relatively short datalogger reading intervals should be used, on the order of 30 seconds or less.

Incremental rigidity evaluations require that a change in test load, and a change in strain, be determined for each load increment at each strain gage level. The evaluations are enhanced if instead of selecting a single value of test load and strain from the last recorded row of datalogger data from which to determine change, the averages of representative test load and strain readings over the hold time are used. Using average values can make the incremental rigidity plots more-interpretable. This is illustrated in Fig. 5.

Fig. 4 presents incremental rigidity relationships determined using averages of representative test load and strain readings recorded during the load increment hold time. Fig. 5 presents IR relationships determined from the same static loading test, but instead uses the single values (average values from a single SG level if the SG level contains more than one SG) of test load and strain from the last recorded row of datalogger data for each load increment. A comparison of Figs. 4 and 5 indicates that using average values smoothed out the IR plots, making them more-interpretable. The comparison also indicates that the “noise,” or scatter, in the IR plots increased significantly after the foundation base started to displace. In addition, it is apparent in Fig. 5 that base displacement resulted in scatter in the single-value IR plots of even the uppermost strain gage levels.
One benefit of the Incremental Rigidity method is that it can be applied to each individual strain gage level to determine specific IR relationships for each level. For comparison purposes, it is recommended that IR relationships for all strain gage levels in a static loading test pile be presented on one plot. In some cases, it may be that IR relationships among various SG levels are similar enough that a composite best-fit relationship derived from multiple SG levels is appropriate.

In other cases, it may be appropriate that unique incremental rigidity relationships be determined and applied on a level-by-level basis. Nonhomogeneous concrete moduli within a drilled deep foundation have been identified by others (Hong et al. 2019). However, this can be true even for what might be considered a uniform pile, such as a closed-end concreted-in-place (“CIP”) pile. Even in apparently uniform piles, concrete modulus (both its initial (zero-strain) value and its strain-dependent relationship) can vary by location along the pile length. In this context, uniform refers to piles whose total cross-sectional area, and/or the respective areas of steel and concrete, is constant by location within the pile.

Variable rigidity in what would normally be considered a uniform pile is demonstrated in the following example. The head-down static loading test pile is an 18-inch-diameter closed-end concreted-in-place pipe pile, having an embedded length of 175 feet, and 15 strain gage levels approximately evenly spaced along its embedded length (including one at the ground surface).
The 15 SG levels were numbered SG 1 through SG 15 consecutively from the ground surface to just above the pile toe. Table 1 presents the measured strains from the 15 SG levels for the maximum applied load.

Table 1. Strain Gage Levels’ Measured Change in Microstrain During Static Loading Test

<table>
<thead>
<tr>
<th>SG 1</th>
<th>SG 2</th>
<th>SG 3</th>
<th>SG 4</th>
<th>SG 5</th>
<th>SG 6</th>
<th>SG 7</th>
<th>SG 8</th>
<th>SG 9</th>
<th>SG 10</th>
<th>SG 11</th>
<th>SG 12</th>
<th>SG 13</th>
<th>SG 14</th>
<th>SG 15</th>
</tr>
</thead>
</table>

Inspection of Table 1 indicates that four strain gage levels measured strains greater than a strain gage level above them (underlined values). In a truly uniform pile, this is a physical impossibility. Conventional practice might perceive these readings in a CIP pile as suspect and ignore the results from these SG levels, diminishing the effectiveness and value of the instrumentation program. In fact, while performing the test, the engineer noticed the apparent unreasonableness of these values, and lamented the seemingly high gage mortality rate. Fig. 6 presents incremental rigidity plots for all strain gage levels in the test pile.

Inspection of Fig. 6 indicates that, as is commonly the case for a concreted-in-place pipe pile, Strain Gage Level 1 located at the ground surface exhibited a different incremental rigidity relationship than other SG levels. By inspection, and evidenced by their positive slopes, it is apparent that SG Level 9 and all levels below it are affected by pile base/toe displacement, and are
therefore non-interpretable. Conversion of strain to internal force at non-interpretable SG levels using IR results in an apparently uniform pile will be addressed in a subsequent section.

To illustrate the effect of using a single incremental rigidity relationship determined from the interpretable strain gage levels and applying it to all SG levels, Fig. 7 presents the IR relationships for SG Levels 2 through 8, and the combined best-fit line through the data points.

![Incremental Rigidity Diagram](image)

**Fig. 7.** Incremental rigidity diagram for those strain gage levels that showed similar IR relationships, illustrating selection of a single best-fit trend line.

To provide comparison to the more-conventional approach of estimating concrete modulus individually (as opposed to determining rigidity, EA), \( E_{\text{CONC}} \) was estimated using the relationship relating it to unconfined compressive strength given in the ACI 318-14 manual (2014):

\[
E_{\text{CONC}} = w_c^{1.533}(f'_c)^{0.5} \tag{Eq. 3}
\]

Where:  
\( E_{\text{CONC}} \) = Concrete elastic modulus, pounds per square inch (“psi”)  
\( w_c \) = Concrete unit weight, pounds per cubic foot (“pcf”)  
\( f'_c \) = Concrete test cylinder unconfined compressive strength, psi

The average compressive strength of three concrete cylinders tested the day before the static loading test was 5,327 psi. Assuming a concrete unit weight of 150 pcf, the ACI relationship yields a calculated concrete elastic modulus value of 4,424x10^3 psi, and a corresponding calculated pile rigidly of 2.03x10^6 kips. Since the strain-dependency of \( E_{\text{CONC}} \) is not accounted for in Eq. 3, a pile’s rigidity so-determined plots as a horizontal line on an incremental rigidity plot (Fig. 7).
The internal force profiles calculated by applying the constant rigidity value determined from $E_{\text{CONC}}$ based on the ACI relationship to all strain gage levels is presented in Fig. 8.

Fig. 9 presents the internal force profiles calculated using the single best-fit strain-dependent incremental rigidity relationship (Fig. 7) applied to all strain gage levels. Internal forces calculated using the constant (not strain-dependent) rigidity value determined using $E_{\text{CONC}}$ from the ACI relationship (Fig. 8) exceed those calculated using the incremental rigidity plots’ single best-fit trend line (Fig. 9) by 15 percent at the maximum strain measured during the static loading test.

Inspection of Figs. 8 and 9 confirms the indication of physically impossible internal force profiles, confirming that the test pile is indeed non-uniform, and that the use of a single incremental rigidity relationship (Fig. 9) is inappropriate. Since the total cross-sectional area and the relative areas of steel and concrete in the concreted-in-place pipe pile were the same at each strain gage level, and the modulus of steel can be considered constant at all locations, the concrete modulus at the associated SG levels must have varied by location within the pile.

Fig. 10 presents the incremental rigidity relationships for Strain Gage Levels 2 through 8 plotted to a larger scale for clarity, with best-fit lines through each individual IR relationship. Inspection of Fig. 10 indicates that the individual IR relationships differ slightly/somewhat from one another. Differences in initial (zero-strain) incremental rigidity values, and therefore initial concrete modulus values, determined from these seven SG levels vary by up to 15 percent.
Fig. 10. IR diagram illustrating individual relationships for SG Levels 2 through 8.

Fig. 11 presents the internal force profiles determined from using the individual strain-dependent incremental rigidity relationships (Fig. 10). Inspection of Fig. 11 indicates that the internal force profiles now appear reasonable, making physical sense in that all internal forces consistently decrease with depth.

DETERMINING RIGIDITIES AT NON-INTERPRETABLE STRAIN GAGE LEVELS

It has been demonstrated that not all strain gage levels in an instrumented static loading test pile necessarily yield interpretable incremental rigidity relationships. Results from interpretable IR relationships can be used to determine rigidities at non-interpretable SG levels.

General

The initial rigidity, steel elastic modulus, and cross-sectional areas of concrete and steel can be used to back-calculate an initial concrete modulus at strain gage levels that yield interpretable incremental rigidity results by the following relationship:
\[(E_{\text{CONC}})_{\text{INITIAL}} = ((E_{\text{A}})_{\text{INITIAL}} - E_{\text{STEEL}}A_{\text{STEEL}})) / A_{\text{CONC}} \]  \[\text{[Eq. 4]}\]

Where:
- \((E_{\text{CONC}})_{\text{INITIAL}}\) = Initial concrete modulus (i.e., at zero strain)
- \((E_{\text{A}})_{\text{INITIAL}}\) = Initial rigidity (i.e., at zero strain) for the composite section
- \(E_{\text{STEEL}}\) = Steel modulus
- \(A_{\text{STEEL}}\) = Cross-sectional steel area
- \(A_{\text{CONC}}\) = Cross-sectional concrete area

At strain gage levels where incremental rigidity analyses do not yield interpretable results, such back-calculated initial concrete moduli can be used to determine initial rigidity by the following relationship:

\[(E_{\text{A}})_{\text{INITIAL}} = E_{\text{STEEL}}A_{\text{STEEL}} + (E_{\text{CONC}})_{\text{INITIAL}}A_{\text{CONC}} \]  \[\text{[Eq. 5]}\]

\((E_{\text{A}})_{\text{INITIAL}}\) is the “b” parameter in Eq. 2. To calculate internal force at non-interpretable strain gage levels, a reasonable strain-dependent rigidity slope (the “a” parameter in Eq. 2) is selected based on results from interpretable SG levels.

Fig. 11. Internal force profiles determined using individual incremental rigidity relationships for each strain gage level.
Apparently Uniform Piles

It has been demonstrated herein that the rigidity of even apparently uniform concreted-in-place pipe piles can vary by location within the pile, owing to the concrete modulus varying by location within the pile. Since only the concrete modulus potentially varies within a pile of uniform cross-sectional areas (of steel and concrete), an alternative approach can be used to apply results from interpretable strain gage levels to non-interpretable levels.

The initial rigidity is determined for all interpretable strain gage levels. These initial rigidity values are plotted against concrete depth (the fluid concrete head to which the concrete at the SG level was subject during curing) for their respective interpretable SG levels (Fig. 12).

![Graph showing initial incremental rigidity values versus concrete depth.](image)

Such a plot often results in a fairly linear relationship between initial concrete modulus and concrete depth for interpretable strain gage levels, sloping from a smaller initial modulus to a larger one with increasing concrete depth. Initial rigidity values determined from non-interpretable strain gage levels deviate from this linearity, often increasing rapidly as strains and therefore back-calculated rigidities are affected by pile-end displacements.

Inspection of Fig. 12 indicates that such a diagram can serve two useful purposes. First, a break in the plot’s linearity can help determine which strain gage levels can be considered interpretable. For this test pile, although Strain Gage Level 9’s initial incremental rigidity value satisfies the...
linear trend, inspection of Fig. 6 indicates that its slope is suspect. Second, extrapolation of the
relationship’s linear portion can provide initial rigidity values to use at SG levels that exhibit non-
interpretable results. Again, to calculate internal force at non-interpretable SG levels, a reasonable
strain-dependent rigidity slope (the “$a$” parameter in Eq. 2) must be selected based on results from
interpretable SG levels. Similarly plotting the slopes of incremental rigidity relationships against
cracking depth may provide similar insights (a thorough review of this response is beyond the
scope of this paper).

APPLICATION TO COMPLEX GEOMETRIES

Proper interpretation of strain measurements in non-uniform drilled foundations often requires
close examination of the incremental rigidity relationships among multiple strain gage levels.
Non-uniform cross-sectional area with depth, often exhibited in ACIP piles and drilled shafts, can
be identified using Thermal Integrity Profiling (“TIP”). Conventional strain measurements’
interpretation involves utilizing semi-empirical correlations of concrete cylinder or grout cube
uniaxial compression strength results to elastic moduli. The pile’s rigidity is then computed from
an assumed cross-sectional area, or from a calculated cross-sectional area using applicable
integrity profiling methods. This conventional procedure can oftentimes result in unrealistic
computed internal force profiles, which results in similarly unrealistic assessment of unit shaft
resistances between SG levels with identifiable differences in geometry. The benefit of IR
analyses not requiring estimation of elastic modulus and cross-sectional area individually at each
SG level, but instead determining a foundation’s rigidity and an internal force-strain relationship
at each SG level, is illustrated by the following case.

A 24-inch-diameter sacrificial ACIP was constructed within a coral stratum for a bi-directional
static loading test. Construction records indicate a large void was encountered approximately 60
feet below the ground surface. The grout volume available on-site during initial drilling was
depleted prior to finishing pile construction. Additional grout was brought to the project location,
and the foundation was subsequently re-drilled (after the originally placed grout had at least
partially set) and re-grouted the full length approximately one hour after completing the initial
construction attempt.

Thermal integrity profiling was used to compute the Effective Average Radii, and thereby develop
a model of nominal pile cross-sectional area versus depth. The void within the coral stratum
resulted in TIP indicating a cross-sectional area increase in the grouted pile from approximately
55 to 72 feet below the pile head. As a result, additional interpretative measures were taken to
estimate the as-built (i.e., post-grouted) cross-sectional area, and associated internal force profiles.

Strain gage Level B1, approximately 60 feet below the pile head, was located within the zone of
the TIP-indicated cross-sectional area increase. SG Level B2, approximately 75 feet below the
pile head, was below the TIP-indicated cross-sectional area increase. The incremental rigidity
relationships for these two SG levels are presented in Fig. 13.

Inspection of Fig. 13 indicates that these two SG levels exhibited good agreement between their
incremental rigidity relationships. Assuming the grout moduli at these two SG levels to be
essentially equal to each other, the agreement between the rigidity relationships indicate similar cross-sectional areas at the two SG levels during the test. This was attributed to the re-drilling

![24-Inch-Diameter ACIP Pile](image)

Potential graph data:

\[
\frac{\Delta Q}{\Delta \varepsilon} = -2.4273 \times 10^{-3} \mu \varepsilon + 4.2009 \times 10^0
\]

Fig. 13. Similar incremental rigidity relationships exhibited by strain gage levels with apparently different cross-sectional areas.

potentially resulting in grout-on-grout shearing (along the re-drilled sidewalls) rather than grout-on-coral shearing (along the increased cross-sectional sidewalls) in this zone. Accordingly, a common IR relationship was applied to both SG Levels B1 and B2 to convert strain to internal force in the pile. The resulting internal force profiles and calculated unit shaft resistances seemed reasonable, which would not have been the case had the IR method not identified similar rigidities at these two SG levels with apparently different cross-sectional areas. The IR method therefore can effectively be applied for improved interpretation of internal force profiles in the complex environment of as-built ACIP geometries and shearing mechanisms in voided or vuggy subsurface materials.

**STRAIN GAGE CALIBRATION FACTORS AND MEASUREMENT UNITS**

It is recommended that, whenever possible, strain be accurately determined at instrumentation levels for conversion to internal force. If incorrect gage factors are inadvertently applied, or if gage factors are unknown (and perhaps a gage factor of unity is applied to readings), strains will not be accurately determined. However, in such cases, the Incremental Rigidity method can still be applied to calculate internal forces at individual interpretable strain gage levels. At an
individual interpretable SG level, the IR method can determine a relationship between applied loads and strain measurements affected by incorrect gage factors, and internal forces can then be correctly calculated by applying that relationship to the affected strain measurements. An additional benefit of this capability is that slight differences in the batch calibrations of certain types of strain gages are accounted for. Similarly, the IR method can be applied to measurements that report alternative units (e.g., digits for vibrating-wire strain gages, volts for resistance-type strain gages) whose relationship is linear with respect to changing strain.

It is noted that in both cases (inaccurate strain measurements due to incorrect calibration factors, or alternate reported units), the relationships determined by the Incremental Rigidity method serve only to convert readings to internal forces at individual interpretable strain gage levels, and are not indicative of the foundation’s structural properties. Accordingly, interpretable results so determined cannot be used to determine rigidities or internal forces at non-interpretable levels.

**CONCLUSIONS**

A valuable objective of statically load testing deep foundations is to determine load-transfer behavior. Integral to this objective is determining internal forces at various locations along the length of the deep-foundation element. These internal forces are usually obtained from strain measurements. Using applied test loads and measured strains, the Incremental Rigidity (“IR”) method determines the relationship between axial rigidity, $EA$, and strain at a strain gage level.

Results from several instrumented static load tests where the Incremental Rigidity method was applied to strain data were presented, and a number of aspects of the IR method were discussed. The elastic modulus of concrete and grout varies with strain; the IR method accounts for this strain-dependent response. The potential error introduced by not accounting for this strain dependency (i.e., using a constant elastic modulus value) was quantified. It was illustrated that relatively small strains can yield meaningful rigidity determinations. Limitations of applying moduli or rigidity values determined from at- or above-grade strain gages to locations throughout the pile length were discussed.

Guidance was provided regarding application of values from strain gage levels that yield interpretable incremental rigidity results so as to obtain meaningful results from strain gage levels that yield non-interpretable IR results. The benefits to IR analyses of averaging test loads and strains was demonstrated. IR analyses of concreted-in-place pipe piles indicate that the elastic modulus of concrete can vary over relatively short distances within a pile, and that load-transfer determinations are improved by evaluating and applying rigidity values for individual strain-gage levels. Results from concreted-in-place pipe piles also indicate that the elastic modulus of concrete may increase linearly with depth. A case history is presented for an augered cast-in-place pile which had an enlarged cross-sectional area over a portion of its length, and the IR method provided estimation of the failure surface through the non-uniform geometry. It was noted that the IR method can be used to determine internal forces at individual interpretable strain gage levels even when strain readings are affected by application of incorrect gage factors, or when only readings of non-strain units are available.
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