WAVE EQUATION ANALYSIS OF PILE DRIVING: METHODOLOGY AND PERFORMANCE

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ABSTRACT

The term WAVE EQUATION is a name applied to a number of computer programs that simulate and analyze impact pile driving according to one dimensional elastic wave propagation theories. The concept was recognized over a century ago, but was not put into practice due to the complexity of the problem and the difficulty of solving it in closed form. Today, discrete mathematical methods and the general availability of electronic computers make the WAVE EQUATION a very practical and indispensable tool for the analyses of pile driving. Questions regarding hammer-pile compatibility, pile driving stresses, and pile load carrying capability can be addressed and resolved by means of simulated models and WAVE EQUATION analyses. Discussed in this article are the ways in which the various components (i.e., hammer, driving system, pile, and soil) are modelled, mathematical details, and a discussion of analysis results. Two case histories will be analyzed by the WAVE EQUATION and results compared with actually measured field performance. Both steel and concrete piles along with air/steam and diesel hammers are considered. The quantities used to evaluate the performance of the WAVE EQUATION are: maximum transferred hammer energy, pile driving stresses, and pile bearing capacity. The GRLWEAP program is exclusively used in these studies.

INTRODUCTION

The basic premise of dynamic pile analysis is very simple: The harder it is to hammer a pile into the ground, the more load it will be able to carry. It was not until the 19th century that quantitative assessments of pile load bearing capability were seriously attempted. Applying Newtonian physics of rigid body impacts, engineers formulated the relationship between a pile's penetration under a hammer blow and its bearing capacity based upon certain fundamental energy considerations. These formulas are known as Dynamic or Energy Formulas. More than 400 of these formulas have been proposed; the most widely used in the United States is probably the simple Engineering News Formula (ENR), published (1) by A. M. Wellington in 1893.

Knowing the pile's set under a hammer blow, the ENR formula suggests a pile working load with an implicit safety factor of 6. Experience, however, indicates that this equation yields actual safety factors as low as 0.5 and as high as 16 or more, when compared to static load tests (2). In the famous Michigan research project (3), it was reported that some of the more elaborate dynamic formulas gave predictions of pile loads between 7 to 30 times the measured value (4). In fact, a survey of the literature shows that, more often than not, dynamic formulas had been found to be grossly inaccurate and unreliable to the extent that it is suggested they be phased out (5) and not be used to determine pile bearing capacity.

Actually, Newton himself had warned against the application of his theory of impact to the problem of pile driving (6). In addition to this basic criticism, the shortcomings of the dynamic formulas are rooted in their simplicity: incomplete, crude, and over-simplified modeling of all components involved. In spite of their noted unreliability, dynamic formulas are still used by some engineers because of their awe inspiring simplicity. This compromise can, however, be very costly.

On the other hand, the observation of blow count or penetration per blow would be a very convenient way to gage the quality of pile foundations. Thus, a rational, reliable dynamic method of capacity prediction based on blow count will continue to be a very attractive alternative to very costly and often difficult to perform static testing. The Wave Equation offers the engineer such a theoretically sound and accurate analytical procedure. This paper provides an insight to the methodology and a performance evaluation of the Wave Equation of pile driving with particular emphasis on the GRLWEAP program.

BACKGROUND

The first solution to the one dimensional wave propagation problem in elastic rods were given by St. Venant (7). He also suggested the applicability of the "wave equation" to pile driving. Isaacs (8) and Fox (9) presented work specifically directed towards the analysis of pile driving. Difficulties in representing the real system, and the tedious mathematical computations involved limited the success of these earlier efforts.

In the 1950s, E. A. L. Smith developed an algorithm and later a computer program code for pile driving analyses (10). It may be that this program was the first application of electronic digital computers in non-military engineering work. In the United States, computer programs based on Smith's numerical solution became known as the WAVE EQUATION. Smith's concept was evaluated by a number of researchers (11) and modifications and improvements were often suggested. Today, the most commonly used wave equation programs are based on either WEAP (12, 13) or TTI (14). Both programs had Federal Highway Administration (FHWA) sponsorship. The most recently released GRLWEAP is a WEAP-based program with a number of special options and enhancements for personal computers.

History of GRLWEAP

The original WEAP (Wave Equation Analyses of Pile Driving) was developed (12) under FHWA sponsorship starting in 1974 after it became evident that existing programs were inadequate for the analyses of diesel pile driving hammers. The WEAP authors had accumulated field dynamic measurements with which the program results were compared. WEAP not only provided the user with an improved hammer model, but also with a simplified data input. A first WEAP update was made in 1981. Research at the University of Colorado (15) into residual stress analyses was incorporated in 1983 and the new version was called CUWEAP. By 1986, advancements in computer technology, new hammer types, and additional research (16), suggested the further update of the program which led to WEAP86 which also incorporated the results of the CU work.

WEAP86 can be executed on both mainframe, or on an IBM PC. The main features of this new version, and the differences between it and the original program had been discussed in various publications (17). After a one year trial period, user feedback along with new research findings were incorporated in 1987 into a slightly revised version called WEAP87, including editorial changes in the documentation. The most recent version is GRLWEAP including preprocessing (GRLINP) and postprocessing (GRLGRF) programs. Among the improvements of GRLWEAP are

- o Blow Count vs Depth (special option)
- o Blow Count vs Stroke (special option)
- o Blow Count vs Capacity (traditional)
- o Pile length up to 1500 ft may be analyzed (traditionally 300 ft)
- o SI or English Units
- o Graphics During Execution for fully automated processing
- o Extensive hammer data file

The minimum requirements, for the PC is a 640 k byte memory and at least one disk drive. However, it is suggested that a coprocessor be used and a printer. Screen graphics would take advantage of the program's many output features.

Modeling Procedures

Improvements that the wave equation offer over dynamic formulas is the ability to realistically model all hammer, cushions, pile cap, pile, and soil components. The so-called lumped mass model is in reality a discrete representation of the linear one-dimensional wave equation

$$\partial^2 u / \partial t^2 = c^2 \partial^2 u / \partial x^2$$

where is the rod displacement at point x and time t. The c^2 value is the square of the wave speed and is equal to elastic modulus divided by mass density. A typical lumped-mass representation of a pile driving system is shown in Figure 1. In the actual modeling process, the size of each mass is determined from the weight of the segment represented. Likewise, a spring would have the same stiffness as the represented element. Coefficients of restitution, round-out deformations, and viscous dampers are also included in the model.

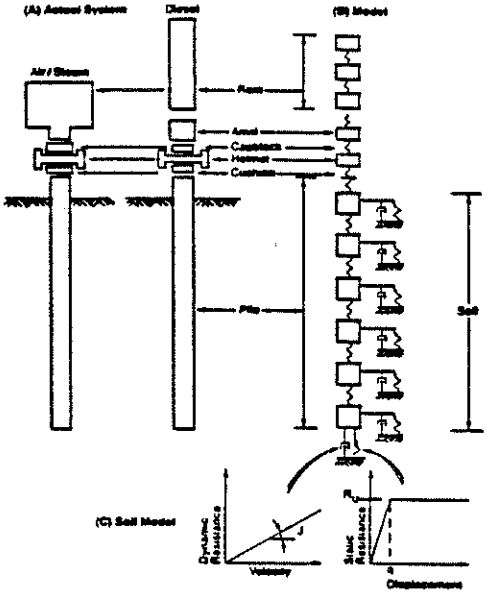

Figure 1: Hammer, Driving System, Pile, Soil Model.

Hammer Model. There are two basic types of impact pile driving hammers: External Combustion (EC), and Internal Combustion (IC). The former encompasses cable, air, steam, or hydraulically powered hammers. The later refers to diesel hammers. Furthermore, both EC and IC hammers may be subdivided into single, or double acting; indicating whether power is only used to move the ram upwards or also during its fall. For EC hammers, the representation is straight-forward: a stocky ram is usually represented by a single mass, the hammer assembly (cylinder, etc.) by two masses and springs.

For IC hammers, the modeling is more involved. The slender ram is divided into several segments, one mass is added for the impact block, and the gas pressure of diesel combustion is calculated according to the Gas Law. The model considers the different effects of liquid and atomized fuel injection. Since the stroke of a diesel hammer is partially dependent on soil resistance, the program may (i) automatically compute the stroke for each resistance, (ii) analyze the same resistance with different strokes, or (iii) analyze different resistances with the same stroke. As part of the GRLWEAP data file, the models of over 200 commonly encountered hammers have been compiled and stored on disk for easy access. Results of a research project on driving system performance (16) were incorporated in the performance parameters of the hammer data file.

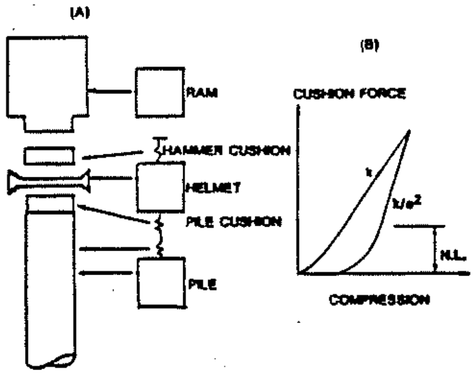

Figure 2: Driving System Model (a) and Cushion Force Deformation Behavior (b) (N.L. - Non Linear Range).

Driving System Model. This model represents the hammer cushion, helmet and pile top cushion (Figure 2a). The helmet is a very compact element that contributes little or no flexibility to the system, so it is treated as a single mass assigned the weight of all driving system parts. Both the hammer cushion and the pile cushion soften the effect of the impact on the ram and the pile and thus behave like springs whose stiffness is calculated from elastic modulus (E), area (A), and thickness (t).

The cushion models also contain a coefficient of restitution to account for energy dissipation and a non-linear spring behavior for low stresses (Figure 2b). GRLWEAP's User's Manual contains extensive tables of helmet weight, modulus, and coefficient of restitution for various manufacturer's suggested driving system components.

Pile Model. To satisfy the basic wave equation, the pile length should be at least ten times its diameter. Piles can be made of timber, concrete, steel, or any combination of these materials. Their cross sectional area may be uniform, uniformly tapered, or step-wise changing.

Any pile can be modeled as a series of masses and springs representing pile segments of approximately 5 ft length. The mass is computed as the product of material mass density, cross sectional area, and segment length. The spring stiffness is the product of the elastic modulus and area divided by segment length.

The user may specify the number of pile segments, or leave it to the program to compute. GRLWEAP can analyze piles with up to 299 elements. Additional features of the pile model include dashpots between pile segments to account for internal pile damping, and nonlinear springs to model splices and slacks.

Soil Model. Soil resistance to pile penetration is represented by both a displacement and a velocity dependent part. At each pile segment below ground level a soil resistance model is assigned, as illustrated in Figure 1. The displacement dependent component can be thought of as an elasto-plastic spring. The deformation at which the plastic behavior starts is called quake (q). For use with the GRLWEAP, it is suggested that a skin quake of 0.1 inch be assigned; the toe quake should be computed from:

$$q\ (\text{toe}) = D/120 \qquad (2)$$

where D is the nominal pile tip diameter in inches. The velocity dependent or dynamic soil resistance is represented by a linear dashpot. It is suggested that a value of 0.15 sec/ft be applied at the pile tip. Valued for skin friction are related to the soil conditions and range between 0.05 and 0.20 sec/ft for non-cohesive and cohesive soils, respectively.

Normally, a series of static pile capacities is analyzed. The program then requires the input of the percentage of the ultimate which is skin friction and the distribution of the friction along the shaft. The user can manually input the skin friction profile, or may choose one of ten distributions already stored as part of the data file. GRLWEAP offers the user the capability to analyze a pile for different input capacities with a constant end bearing, constant skin friction, or the traditional method of resistance increasing for both friction and end bearing. Furthermore, if the user inputs a unit friction and end bearing as a function of depth then the program calculates stresses, transferred energies and, most importantly, blow count at up to 10 penetration points. The option for analyzing multiple hammer blows for residual stresses is also available to the GRLWEAP user.

Summary of Input Parameters The following is a general summary of the different parameters needed for the execution of a wave equation analyses:

Hammer: model and efficiency.
Hammer and/or Pile Cushion: area, thickness,

elastic modulus, and coefficient of restitution.
Helmet: Weight including all cushion materials and any inserts.
Pile: area, elastic modulus, and mass density, all as a function of pile length.
Soil: total static capacity, percent skin friction and its distribution, quake and damping constants, both along the shaft and at the toe or alternatively a soil analysis as a function of depth.

Numerical Analyses. In the beginning of the analysis, all driving system, pile, and soil components are assumed at rest and under zero stress condition (this assumes that no residual stress analyses was elected). The ram is assigned an impact velocity computed from the fall height and hammer efficiency. During a small time increment, the ram moves a short distance compressing the hammer cushion and exerting a force (computed from spring stiffness and deformation) on the helmet. The acceleration of the helmet segment, computed from the applied forces, is integrated to yield the change in velocity and displacement for the time step. With the new displacement values, updated forces in all the springs can then be computed. Similar computations are made for each pile segment. For segments with soil resistance, segment displacement and velocity are used to compute resisting forces, which are also included in the force equilibrium equations for that element. Once forces, accelerations, velocities, and displacements for all elements are computed, the analyses is repeated for the next time step starting from the current parameters.

The user has the option to specify the total length of time the analyses should be carried out, or the program will automatically stop when the pile rebounds a short distance. At the end of the analyses, pile permanent set (the inverse of blow count) is calculated by subtracting a weighted quake average from the maximum computed toe displacement. Thus, for the given static capacity analyzed, a pile set under a hammer blow is established; the stresses along the pile shaft are calculated from the deformations of the pile springs. The analysis may be repeated for another static capacity and a bearing graph is established (see example). The program's actual algorithms are more complex than described but for the purpose of general understanding of the process, this description seems adequate.

In practice, the wave equation analyses is employed to deal with one, or both of the following questions:

1. Given a complete description of hammer, cushions, pile, and soil; can the pile be safely driven to the required bearing capacity?
2. What is the static bearing capacity of the pile given the pile driving or restrike blow count?

In the first problem proper equipment selection or the pile design can be verified before actually going in the field. Calculated stresses in the pile should remain safely below yield or compressive/tensile strength of the pile material. The calculated blow count should be economical for production.

In the second situation, the pile must be driven to that blow count that corresponds to the required ultimate pile capacity. The ultimate capacity must be greater than the design load. Usual safety factors are at or above two.

It should be noted that accurate predictions of long term pile bearing capacity are only possible if the pile is tested during a restrike. The time between initial installation and retap depends on the soil's speed of strength change after driving.

WAVE EQUATION ANALYSIS PROCEDURE EXAMPLES

To illustrate the applicability, and to evaluate the performance of the wave equation analyses, two specific examples will be investigated. Analyses results will be compared to actual field dynamic measurements, or full scale static load tests.

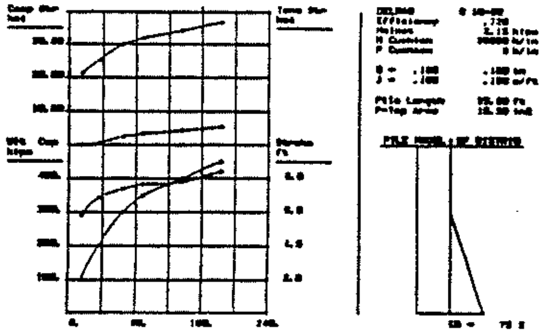

Figure 3: Bearing Graph, Example 1.

EXAMPLE 1

Situation: The existing subsurface conditions at a proposed construction site dictated the use of driven piles as a foundation system. The soil conditions could generally be described as a layer of fill, loose to firm alluvial silts and residual soil overlaying partially weathered rock. The overburden thickness varied from 20 to 40 feet across the site. It was decided to use 12X53 steel H-piles with lengths between 25 and 45 ft. The proposed hammer was a Delmag D 16-32 open end diesel hammer; to be used on fuel setting No. 3. The driving system would consist of aluminum and conbest sandwich and a helmet of 2.15 kips. Each pile was designed for an ultimate static bearing capacity of 400 kips.

Required: Given the above information, can the proposed hammer safely drive the pile to the required capacity? What is the expected driving resistance and pile stresses?

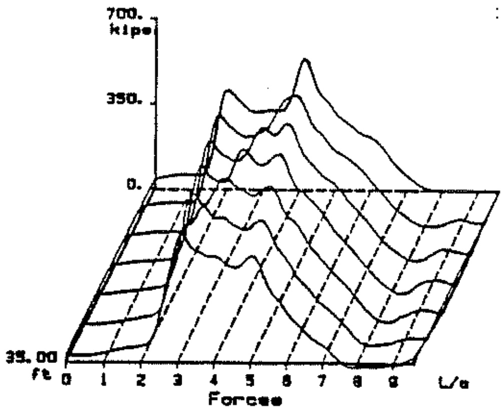

Figure 4: Forces in Pile vs Time for Example 1

Solution: A wave equation analyses using the GRLWEAP program was performed to evaluate the problem above. An average pile length of 35 ft was used. Standard soil, and driving system parameters were input. The stored hammer model, along with its suggested hammer efficiency were elected. A summary of the analyses inputs, along with results plotted outputs are shown in Figure 3. The results are presented in the form of a bearing graph relating pile capacity, driving stresses, and hammer stroke to driving resistance. A GRLWEAP generated plot of force histories for seven pile segments is shown in Figure 4. It was found that in order for the pile to develop a capacity of 400 kips, the blow count should be 11 blows per inch with a ram stroke of 7.8 ft and a transferred energy of 13 kip-ft. At this driving resistance, the pile compressive driving stresses are expected to average 34.4 ksi.

During pile driving, dynamic measurements and analyses (18) of pile top force and velocity using the Pile Driving Analyzer^{TM} (PDA) and CAPWAPC method were performed on several piles as part of a construction quality control program. In the field, the PDA computed maximum pile top compressive stress, transferred energy, and Case Method pile capacity for every hammer blow. Also measured by the PDA was the hammer rate of operation in blows per minutes, which was converted to ram stroke assuming a free fall condition with a small correction. Of those piles driven to between 10 and 12 BPI while the hammer was on fuel setting No. 3, eight were monitored with the PDA. GRLWEAP predictions of pile capacity, compressive stress, transferred energy, and ram stroke at a driving resistance of 11 BPI, were compared to the average field measured data on these eight piles.

Correlations between GRLWEAP predicted, and field measured pile driving dynamic variables fell within expected limits of accuracy. The average measured pile capacity was 428 kips (7% difference), hammer stroke 8.2 ft (5%) transferred energy 15 kip-ft (15%), and pile compressive stress 29 ksi (-18%).

EXAMPLE 2

This example illustrates the effectiveness of the wave equation analyses in assessing the static bearing capacity of a previously driven pile. Approximately three days after initial installation, the pile was restruck with a pile driving hammer, and was statically load tested a few hours later. The restrike driving resistance was evaluated by GRLWEAP for pile load capacity prediction, which was then compared to the actual measured value.

Under consideration was a 12-inch square 60 ft long prestressed concrete pile. The top 13 ft of soil consisted of stiff to hard red-brown silts and clays with N-values in the upper 20s. The next 35 ft contained stiff to hard fine sandy silt, with N-values increasing from 15 to 40 with depth, under which a hard weathered rock was encountered. Driving and restriking were accomplished with a Conmaco 65E5 single acting air/steam hammer. Standard hammer cushions and pile cap were used. Ten 3/4" thick sheets of plywood were employed as pile cushion. The final penetration and driving resistance were reported to be 45 ft, and 5 blows/inch, respectively. The restrike consisted of five hammer blows that resulted in a pile net set of half an inch (an equivalent of 120 blows per foot).

Table 1: Numerical Bearing Graph Results, Example 2.

EXAMPLE 2, STEEL PILE, DIESEL HAMMER

No.	Ultimate Capacity kips	Max C. Stress ksi	Max T. Stress ksi	Blow Count bl/ft	Stroke ft	Energy k-ft
1	100.0	21.469	.000	15.3	5.80	13.23
2	200.0	25.289	.691	35.5	6.87	12.62
3	300.0	30.086	2.380	64.3	7.41	12.80
4	350.0	32.062	3.463	88.4	7.68	13.04
5	400.0	34.387	4.450	136.0	7.81	12.97
6	450.0	36.956	5.481	183.1	8.43	14.06

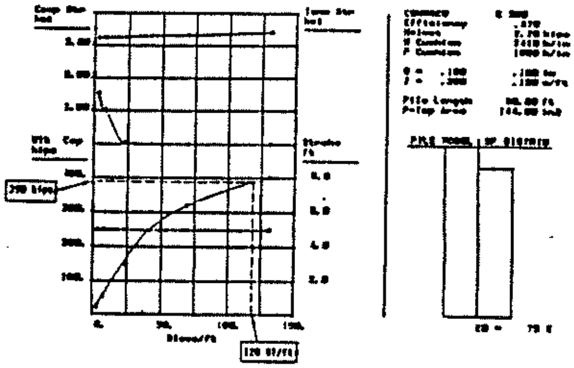

Figure 5: Bearing Graph, Example 2.

A summary of the analyses input variables is presented in Figure 5, along with a Bearing Graph relating pile capacity and driving stresses to blow count. This results is also given numerically in Table 1. At 120 blows/ft GRLWEAP predicted a pile capacity of 390 kips.

A few hours after it was restruck, the pile was subjected to a static load test. The pile was loaded to 414 kips in 40 kip increments at 10 minutes intervals. The pile top load versus set were plotted, and the resulting graph interpreted according to Davisson's criteria (18) for failure load determination; which was found to be 410 kips. The error between GRLWEAP predicted and measured ultimate static pile capacity values was only 5 percent.

CONCLUSION AND RECOMMENDATIONS

The background and development of the wave equation analyses of pile driving, the modeling of hammer, driving system, pile, and soil, and the numerical procedure were discussed. The GRLWEAP program history, methodology, capabilities, and performance were emphasized. The applicability of the analyses in predicting pile stresses and capacities was demonstrated by two examples. Correlations between GRLWEAP predicted and field measured quantities showed very good program performance.

The availability of micro-computers, and the automated program execution procedures do not relieve the engineer from the responsibility of checking his results. All User Manual suggested input values represent an average behavior, it is the engineer's responsibility to make sure that these conditions apply to the job at hand. This is particularly true for the hammer itself as identically rated hammers have often been observed to vary by up to a factor of two in actual performance which can lead to large errors in any dynamic method if not properly recognized. Sensitivity studies by varying the least known parameters should be performed. Analyses results may be verified with actual field measurements which will confirm, or provide a basis for input assumptions corrections. It should be remembered that the results from any computer program are only as reliable as the input data.

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