DYNAMIC STUDIES ON THE BEARING CAPACITY OF PILES

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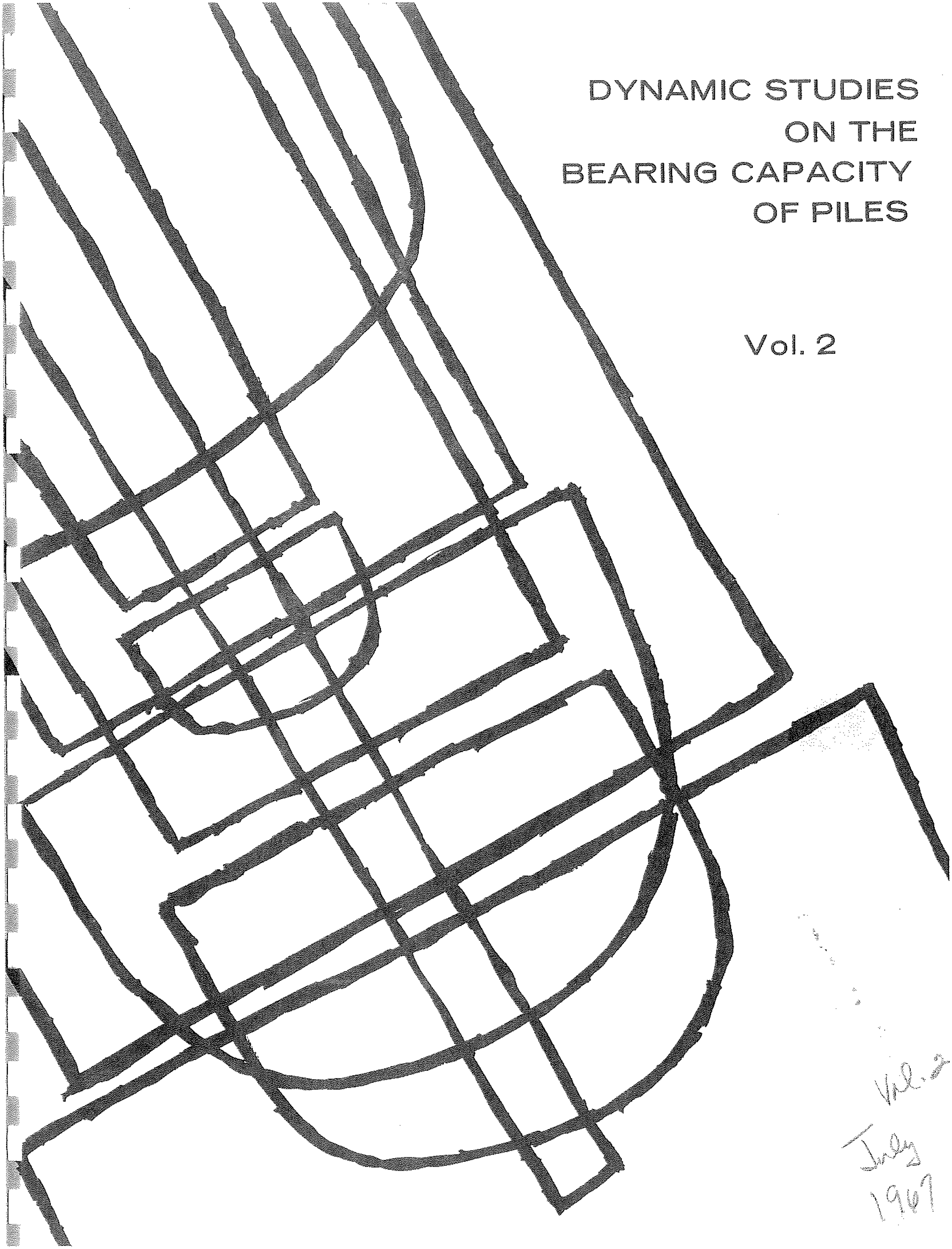

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DYNAMIC STUDIES ON THE BEARING CAPACITY OF PILES

Final Project Report

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CHAPTER 1

INTRODUCTION

The primary purpose of the project was to study, experimentally, the proposed simplified theory. The results of this effort are reported in Volume 1 of this report. Considerable additional effort was devoted in other directions in support of the primary activity.

In the course of searching for experimental data in the literature an extensive Bibliography was compiled. It is included in the Appendix of this Volume. The basis of all of the pile formulas was examined and a review of the results of this study is reported in Chapter 2.

A computer program was prepared to simulate a pile during driving. The pile was modeled by lumped masses interconnected by springs. In this manner all parts of the driving apparatus could be included in the model and soil forces applied along the pile. An initial velocity is then imparted to the upper mass and the motion of the system can be expected to duplicate the dynamic pile behavior. The primary difficulty with such a system lies with the inadequate knowledge of pile-soil interface forces. A correlation was made between dynamic field measurements and the mathematical model results. The effect of a change in dynamic parameters was studied. This work is presented in Chapter 3.

The pile dynamics problem was also approached treating the pile as a continous elastic rod. This problem becomes quite difficult because of the complicated boundary conditions along the rod. Only a beginning was made with this approach. The results are given in Chapter 4.

CHAPTER 2

LITERATURE STUDY

A portion of the project was concerned with a literature investigation. The primary purpose of this work was to obtain experimental data which could be of use in verifying any theory which might be proposed. A bibliography of the papers reviewed is given in the Appendix. A review of all dynamic driving formulas was also made and will be discussed briefly. A more detailed discussion is found in reference 2.1.

2.1 Pile Formulas

Three general approaches have been used traditionally in predicting the bearing capacity of piles. They can be classified under the general headings: static formulas, dynamic formulas and methods using impact and wave theory.

The static formulas all rely on information regarding soil characteristics. Thus, based on investigation of soil samples the attempt is made to predict the friction strength between the pile wall and the soil. Also, prediction of point resistance is made. From this information pile capacity is predicted. It should be emphasized that the static formulas depend on a knowledge of soil behavior taken from the science of soil mechanics. They have their use in design but in cohesive soils where strength development is slow and measurements made during initial driving are unreliable.

There are many dynamic formulas in use today. They are based on both empirical and theoretical considerations and are obtained from energy relationships in which certain assumptions are made. The many and varied assumptions which have been made have led to considerable controversy. However, some assumptions must be made to give the formulas practical usefulness. The dynamic equations will be discussed with the primary goal of emphasizing the physical meaning of the assumptions.

There are only five basic types of dynamic pile-driving formulas. All of them can be represented by the formula

$$Wh = R_d S + Q \qquad 2.1$$

where R_d is the dynamic resistance, h is the height of fall of hammer, in inches, S is the penetration of pile per blow, in inches, Q is all of the energy losses at the time of impact and W is the weight of striking parts of hammer. The basis of this equation is best seen from the following discussion of the work diagrams shown in Figure 2.1. The diagram in Figure 2.1(a) represents the work expended in moving the pile a distance S against a uniform resistance R'. If no energy losses occurred, this work would be equal to the energy available at the time of impact. This diagram is not representative of practical conditions because there will always be an energy dissipation due to temporary elastic compression in the pile and surrounding media, and in all likelihood, varying resistance to penetration. Figures 2.1(b) and 2.1(c) consider elastic compression and two types of varying resistance. The difference between ideal and actual pile behavior is apparent by comparing the shaded area of Figure 2.1(a) superimposed on Figures 2.1(b) and 2.1(c). The resistance in Figure 2.1(b) is similar to the resistance behavior in sand. The resistance increases as the sand is compacted by the moving pile. In this case, the resistance increases with penetration depth until it reaches a value of R". In Figure 2.1(c), the behavior is similar to that expected in a clay soil. The initial value of the resistance (R"') will be high, but will decrease with depth of penetration to a constant value of R". The resistance R" can be related to the resistance R' for the ideal case by a numerical constant C".

$$R'' = C'' R' \qquad 2.2$$

where R" is the final resistance under one hammer blow, C" is the coefficient dependent upon resistance characteristics, and R' is the average value of a variable resistance to penetration under one blow. The value of C" depends on the characteristics of the resistance force between the pile and the soil. If the resistance behavior is like that of Figure 2.1(b), the value of C" would be more than one. Similarly, C" would be less than one for a resistance behavior shown in Figure 2.1(c). The triangular area SBD represents the energy loss due to elastic deformation. The temporary elastic compression at impact, which causes the energy loss, is represented by the distance SS'. From this discussion it is apparent that the equation,

$$Wh = R'S \qquad 2.3$$

must be modified to account for energy loss (adding a value Q) and variable resistance (multiplying R's by C".) The equation in modified form is:

$$Wh = C'' R'S + Q \qquad 2.4$$

These work diagrams could be used as an approach to determining the dynamic bearing capacity of a pile. The disadvantage to using such an approach is that accurate field measurements of the penetration and resistance are required.

Cummings (Ref. 2.1) classified all dynamic pile equations into five types. A.M. Wellington's equation, the Engineering-News formula, was developed from a work diagram similar to that discussed above. This is one of Cummings' five general types of equations. Wellington suggested an energy loss of $0.1R_d$. This formula is expressed as follows:

$$Wh = R_dS + 0.1\ R_d \qquad 2.5$$

Using the value k instead of 0.1 to account for different energy losses with different driving conditions, as previously done for the Engineering-News formula, results in the equation:

$$Wh = R_dS + kR_d \qquad 2.6$$

where 2k is the total rebound of pile hammer.

This equation is in error because there is no valid basis to justify this term.

The second type of dynamic formula given by Cummings is one in which Wh_o represents the lost energy.

$$Wh = R_d S + Wh_o \qquad 2.7$$

where h_o is the maximum height of fall of hammer at which penetration per blow is still zero ($S = 0$). The units are in inches. This formula has been attributed to F. Kreuter but it was published by G. J. Morrison, in 1868. It has been unsatisfactory because it has been impossible to obtain consistent values of h_o.

The Weisback formula can be obtained from Cummings' third basic type of dynamic pile-driving formula in which the energy loss is caused by temporary elastic compression of the pile.

$$Wh = R_d S + \frac{R_d^2 L}{2AE} \qquad 2.8$$

where L is the length of pile as driven in inches, A is the average cross-sectional area of pile and E is the modulus of elasticity of pile.

This equation neglects other energy losses and is based on the assumption that the dynamic energy loss can be computed by static theory. The last term in Equation 2.8 represents the potential energy of strain in a compressed strut subject to a static load of amount R_d at each end. In the first place, it is well known that the elastic compression under impact is something entirely different from the elastic compression due to a static force. In the second place, the resistance is scarcely ever applied entirely at the pile point. There is usually some resistance along the sides of the pile and in many cases practically all of the resistance is on the sides and the point resistance is negligible. Accordingly, it is not to be expected that the energy loss due to temporary elastic compression can be computed with any reasonable degree of accuracy by means of an expression taken from static theory, without modification, for use in a dynamic problem.

If the energy loss is attributed to impact, which assumes that the pile problem is considered to be an impact problem which can be solved by Newton's impact theory, the fourth type of dynamic pile-driving formula will result.

$$Wh = R_d S + Wh \frac{P(1-n^2)}{W + P} \qquad 2.9$$

where n is the coefficient of restitution, and p is the weight of pile. If n equals zero (assuming perfectly inelastic impact), the formula will become Eytelwein's formula, published in 1820. If n equals one (assuming perfectly elastic impact), the formula will become the Sanders equation.

It is a fallacy in the belief that a dynamic pile-driving formula can be formulated using Newton's theory of simple impact. It is apparent from Newton's own words that he did not intend his theory to be applied to pile driving. The theory is applicable to all elastic bodies" ... except where parts of the bodies are damaged in the collision or where they suffer some such extension as occurs under the strokes of a hammer." (Ref. 2.2) It is further seen from a description of the experiments made by Newton, from which he formulated his theory, that the impact experienced in pile driving is not of the same character as the impact realized in his experiments. Newton made the following statement in his explanation of his third law concerning impact. Two colliding bodies will react according to the rules of behavior provided"... the bodies are not hindered by any other impediments." (Ref. 2.2). This statement, along with the fact that there was no restraint which caused elastic distortion in his experiments, rule out pile driving as a problem which can be solved using his theories on impact since the soil surrounding the pile offers resistance. Pile driving is more than a two-body problem because of the constraints introduced by the surrounding media. Therefore, Newton's impact theory, using the coefficient of restitution cannot be applied to pile driving because it is restricted to two-body problems.

The "complete" pile-driving formula, is Cummings' fifth type of equation.

$$Wh = R_d S + Wh \left(\frac{P(1-n^2)}{W + P}\right) + \left(\frac{R_d^2 L}{2AE} + \frac{R_d^2 L'}{2A'E} + K\right) \qquad 2.10$$

where L', A', and E' are defined the same as L, A, E except that they refer to the driving cap, and K is the energy loss caused by the temporary elastic compression of the soil. This equation is invalid for the same reasons that Equations 2.8 and 2.9 are invalid. It is deceiving in that it appears to have taken everything into account, but in actuality some of the energy losses are considered twice. This results from including the two types of losses (impact and elastic) in one equation.

Other dynamic formulas can be similarly discussed. It will be found that they can be classified as one of Cummings' five basic types. A formula which is placed in a certain group will have the same limitations as that set.

The accuracy of the many pile formulas has been the subject of considerable study. Since they have generally fallen into disrepute their unreliability will not be discussed further. An extensive investigation of a large number of common pile formulas was conducted in connection with the project and is reported in reference 2.3.

2.2 Longitudinal Impact and Wave Theory as Applied to Pile Driving

A pile, during driving, acts not as a body which is instaneously compressed at impact, but as a structure which transmits response waves (stress waves, velocity waves, etc.) throughout its length. St. Venant and Boussinesq were among the first to examine the problems of longitudinal wave transmission and impact. This was over 100 years ago. They examined two types of longitudinal impact. The first was that of a rod struck longitudinally at one end and fixed at the other. The second case considered a rod which was similarly struck, but free at the opposite end. The results obtained from the analysis of the rod restrained at one end can be applied to a pile since it encounters resistance from the surrounding media.

Apparently Isaacs (Ref. 2.4) was the first to associate wave action with pile driving. His method of analysis was similar to that followed by St. Venant and Boussinesq. He used boundary conditions which were more in line with the physical characteristics of the pile. Other work has since been done by Fox, (Ref. 2.5) Glanville, Grime, Fox and Davies, (Ref. 2.6) and others.

If it is assumed that the rod with a fixed end, examined by St. Venant and Boussinesq, is a pile, their assumptions are:

1. The sides of the pile are free and there is no side friction which would affect the stress waves running up and down the pile.

2. Stress waves in the hammer may be neglected.

3. There are no flexural vibrations of the pile.

4. The pile behaves as a linearly elastic rod.

5. The hammer strikes directly on the head of the pile and the surfaces of contact are two ideal smooth parallel planes.

6. The lower end of the pile is fixed.

The theory does not include the effect of dissipation of energy due to propagation losses in the pile. The validity of these assumptions, as applied to pile driving is as follows:

The assumptions of neglecting skin friction and propagation losses are on the safe side. This is because the friction forces along the side of a pile tend to reduce the amplitude of the stress waves causing lower stresses in the pile. Similarly, propagation losses tend to reduce the stresses. Therefore, these assumptions will cause the theoretical stress state to be higher than in the actual pile.

The assumption of considering the hammer as a rigid body can be regarded as a realistic one because it is usually a short heavy block of steel.

It is always possible for a pile to fail by flexural buckling under a static or dynamic load. However, the bending stresses and strain energy of bending calculated to occur during driving, using values for the eccentricity of the hammer blow and misalignment of the pile which are within the allowable range for an actual pile driving job, are small when compared to other stresses and the energy which the hammer provides at impact.

Assumption four is valid for piles which are of one section and one material. This assumption is not valid for composite piles or piles which are made up of two or more separate sections.

A cushion or driving cap is provided between the hammer and pile head in practically all driving cases. These items reduce the stresses in the pile. Using assumption five, which neglects the influence of these objects in the analysis, will result in higher theoretical stresses than in the actual case.

The pile point does not remain fixed. In practice, the point of the pile will be resisted by a force which is more or less elastic in character. The elastic reaction will induce smaller stresses in the pile than the force which is required to cause zero displacement at the point.

Isaacs approached the pile problem in a manner similar to the method used by St. Venant and Boussinesq on the impact problem. The difference was the assumptions made in the boundary conditions and initial conditions. Fox (Ref. 2.5) presented the main points of Isaacs' theory in a paper concerned with stresses in piles during driving. A review of this article is included in reference 2.3.

The assumptions made by Isaacs were similar to those listed above except:

1. It is assumed that the capblock packing remains at a constant thickness after the maximum stress is reached, unless the stress caused by a reflected wave from the foot exceeds this maximum value. If this occurs the packing is further compressed until a new maximum stress is reached. This process is repeated for other reflected waves.

2. It is assumed that the toe of the pile will not move until the compressive stress at the point of the pile is equal to p. When this stress equals p, the toe will begin to move until the velocity of the point again equals zero. The compressive stress at the point is assumed to remain at the value of p during movement. When movement of the toe stops, the stress will become smaller. Movement can again occur if the reflected waves from the head of the pile cause the compressive stress to equal p.

Returning to the rod with the fixed end, the theory of St. Venant and Boussinesq gives the following equation for the maximum compressive stress which occurs at the fixed end when the ratio of the striking body to the weight of the rod is less than five:

$$p_{max} = \frac{2EV}{c} \quad (1 + e^{-2(P/W)}{}_L) \qquad 2.11$$

where p_{max} is the maximum compressive stress, E is the modulus of elasticity of the rod, c is the velocity of stress in the rod, V is the impact velocity of striking body, P is the weight of rod, and W is the weight of striking body. The stress calculated from this formula will be considerably higher than the maximum stress which occurs in the actual pile for reasons previously discussed.

In reference 2.5 and 2.6 it was desired to obtain equations for stress response which gave a closer approximation to actual stresses.

On the basis of these modified assumptions, a complete solution of the differential equation was developed. This complete solution includes long and complicated mathematical expressions so that its use for a practical problem would involve laborious numerical calculations. In order to avoid this wherever possible the engineers of the British Building Research Board developed approximate solutions for certain specific conditions.

For one of these approximate solutions the maximum stress in the pile is given by

$$p_{max} = \frac{EV}{c} \quad \frac{W}{P(1+\frac{E}{TL})} \qquad 2.12$$

where T is the stiffness constant of cushion block, and L is the length of pile.

Because of the approximations involved in the derivation of Equation 2.12, its use is limited to heavy hammers, soft cushion blocks and short piles that are driven against a practically rigid bottom. The approximate accuracy of Equation 2.12 can be determined from the expression

$$\frac{P}{3W(1+\frac{E}{TL})} \qquad 2.13$$

which represents the ratio of terms neglected to terms retained. The smaller the value of the ratio the greater will be the accuracy of the approximation.

The articles which have been mentioned in this discussion are not the only ones which examine the pile with regard to longitudinal impact and wave response theory. Further information is given in References 2.7, 2.8 and 2.9.

The dynamic response studies discussed above have as their goal the determination of stresses in the pile during driving. They do not attempt to contribute information on bearing capacity.

Forehand and Reese (Ref. 2.10) have made use of Smith's analysis in their investigation of the possibility that the wave equation can be used to predict the ultimate bearing capacity of a pile. In this study they attempted to establish values for the ground quake, ground damping, and the distribution of side and point resistance for certain types of soils. This was accomplished by the correlation of pile driving records with load test results given in published reports. They conclude that additional research is necessary, but that the wave equation method of analysis promises to become an accurate and general method to be used in conjunction with other factors for predicting the ultimate bearing capacity of a pile from its dynamic behavior under the last hammer blow for all combinations of driving equipment, pile, and types of soil.

CHAPTER 3

LUMPED MASS PILE ANALYSIS

It is possible to use the wave equation to analyze a pile during driving. This approach has great potential, but the complicated boundary and initial conditons cause the equations obtained to be of such a complex nature that their solution is difficult in most practical cases. To arrive at a solution which can be of practical use, or sometimes, to obtain a solution, certain assumptions must be made to simplify the problem. Smith (Ref. 3.1) has presented a method of analysis which attempts to overcome the difficulty in the mathematics. The formulation of the problem by Smith is such that a numerical solution to the wave equation is obtained. If the finite difference relation for the wave equation is compared to the formulas of Smith, it is seen that they are one of the same. This fact has been shown in Reference 2.3.

In Smith's analysis, the pile is divided into a series of finite elements. Each element has a given weight and stiffness. Using these values, a mathematical model is constructed composed of masses and springs. The additional pile components (ram, capblock, pile cap, cushion, core or mandrel, etc.) are similarly represented. The force-displacement characteristics of the springs can be defined mathematically allowing them to portray the actual physical behavior of the pile as closely as possible. Resisting forces can be introduced into the problem to represent the behavior of the surrounding media. By giving the ram an initial velocity, the time history of the pile motion can be studied. Figure 3.1 shows a sketch of the model used for the pile. In reference 2.3 a complete discussion is given of the method used.

The wave equation, solved by a numerical approach, shows promise of being a possible means by which a relationship between the dynamic and static resistance of the surrounding media can be obtained. From this relation, the ultimate bearing capacity could be predicted. It also provides a convenient tool for studying the effect of the variation of dynamic parameters on the response of the pile.

A computer program was developed based on the analysis of Smith, and recommendations made by Samson, Hirsch, and Lowery. (Ref. 3.2). This program was used to simulate pile behavior so that the effect of certain parameters on the dynamic response of the pile could be investigated.

The computer program was used to simulate the response of a pile on which selected measurements were made. The results are used to demonstrate the possibility of correlating field measurements and computer results, and to show certain parameter affects.

The pile investigated was a 12-inch steel test pile located at the interchange of the Willow Freeway and the Clark Freeway in Cleveland, Ohio. It is referred to as Pile No. 113 North Pier in Table 5.5, Volume 1 of this report. It had a hollow circular cross section with an area of 9.82 square inches, and a total length of 63.5 feet, of which 53 feet was below the ground surface.

The hammer used was a Vulcan "0" single acting steam-air-open type. It had a rated striking energy of 24,375 ft-lbs. The weight of the ram is 7500 pounds.

It was not possible to obtain the stiffness of the capblock and the weight of the pile cap. Realistic values were assumed for these unknowns after an examination of the literature. The pile cap was assumed to have a weight of 700 pounds. The stiffness of the capblock was taken to be 2,000,000 pounds per inch. In Figure 3.2, the actual pile and the mathematical model assumed are shown.

The acceleration and strain at a point four feet below the top of the pile were electronically recorded for the last hammer blows on the pile. The parts of the response which were of interest are shown in Figures 3.3 and 3.4. The acceleration curve was graphically integrated to obtain the velocity curve shown in Figure 3.5.

The pile driving log and load test results recorded by Ohio Department of Highways personnel were obtained for this pile. The pile driving record for the last four feet is given in Table 3.1. The exact value of the set at the time the electronic measurements were taken is not known more exactly than the average value. A load-settlement curve is given in Figure 3.6. The failure load as defined by the Ohio Department of Highways, obtained from this test was ninety-nine tons.

The results from the computer program were compared to those obtained in the field to see if the program gave similar results. Values for the soil parameters, quake, Q as shown in Figure 3.7, and damping, J, for point and J' for side, along with a percentage of side resistance, were assumed using values recommended by Forehand and Reese (Ref. 2.10). The values were used as follows:

J = 0.15 — Note: The relation recommended by Forehand and Reese between side and point damping was assumed: J' = (1/3)J.

J' = 0.05

Q = 0.1

31% of the ultimate resistance is side resistance.

Taking different values for the ultimate resistance, R_u in Figure 3.7, and obtaining the corresponding set in inches per blow, a curve can be shown (Figure 3.8.) Once the curve is obtained, the ultimate resistance corresponding to the set of the last blow on the pile can be determined from the curve. This value corresponds to the static resistance of the pile. It was found that the pile had an ultimate resistance of 115 tons when the set was 0.44 inch per blow. This value compares closely to the value of 99 tons found in the load test particularly since the 99 tons value is not the ultimate load but the Ohio Department of Highways yield load.

The velocity, strain, and acceleration response obtained from the computer solution with an ultimate resistance of 200,000 pounds and a set of 0.598 inch per blow, were compared to the respective responses obtained in the field (Figures 3.9, 3.10, and

3.11). This set is slightly higher than the indicated set from the pile driving record. It is not known exactly what the set was at the time of recording and the average value of set near the end of driving did show fluctuation between 0.545 and 0.444 inch per blow, and since these are preliminary results, and certain pile component characteristics are not definitely known the results are within the limits of the accuracy of the measurements.

Upon comparison, it was found that the electronically recorded reaction curves lagged behind the computer obtained responses. It should be noted that the response corresponding to the segment containing the point at which the electronic measurements were made is compared to the electronically measured behavior. A likely explanation is that the capblock had a non-linear stiffness. That is, at first the stiffness was small but as the capblock was compressed the stiffness increased. This type of capblock would cause an initial wave to preceed the main response wave. If the electronically recorded behavior curves are shifted in time so that the lag is eliminated, close agreement is obtained.

In the strain diagram the computer obtained behavior curve intially agrees closely to the response curve found electronically in the field. However, they separate after approximately nine milliseconds with the compressive strain increasing for the numerically obtained response and decreasing for the electronically measured behavior curve.

In both the velocity and acceleration diagrams, it was found that the peak response due to the initial impact determined by the computer solution was considerably higher than the indicated peak found electronically. There are several possible explanations for this. The energy at impact could have been smaller than that given by the manufacturer. The values assumed for the unknown pile cap weight and, or, capblock stiffness could have caused the difference in the peak values. A non-linear stiffness could have reduced the energy available for the main response wave recorded.

A possible explanation why the behavior curves found numerically and electronically for the strain were similar in the beginning, and yet the peak response due to initial impact in both the velocity and acceleration diagrams varied considerably for the computer based solution and electronically measured response is that the segment (mass) points have the same relative movement no matter what the initial peak velocity and acceleration is.

The damping factors were changed to try to get the strain diagram for the computer solution to agree with the electronically recorded response in the area where the two separate (after nine milliseconds). It was found that with an increase in damping the curves were further apart, but with a decrease in the damping factors the strain found by the numerical solution began to decrease in the area of disagreement. It should be noted that the side and point damping were always increased or decreased in the same proportion. The two curves showed good agreement when the damping factors were very small ($J = 0.015$, $J' = 0.005$), but the set had increased considerably. The permanent set was not determined for this case, but it is known that it is over 0.83 inch per blow and is probably over 1.0 inch per blow. The results from which these conclusions are drawn are shown in Figures 3.12 to 3.14. It should be noted that the results obtained by electronic measurements in the field are shown on these graphs even though the permanent sets are not similar. The respective electronically obtained curves are included with each response diagram given in this paper to serve as an aid in comparison.

Using these new values for the damping factors (J = 0.015, J' = 0.005) and the same value for the quake (Q = 0.1) and ultimate resistance distribution (side resistance 31% of the ultimate resistance), the value for the ultimate resistance was changed to reduce the set. The set versus ultimate resistance curve for this case is shown in Figure 3.15. It is seen from this curve that the ultimate resistance corresponding to a set of 0.44 inch per blow is 156 tons. This is high compared to the failure load found from the load test (99 tons). Comparing the strain curve for this case obtained from the results having the closest set to 0.44 inch per blow, it was found that the curve found from the computer solution once again diverged from the electronically recorded curve above nine milliseconds (Figure 3.16, set = 0.48 inch per blow). With this unsatisfactory result, it was decided to decrease the ultimate resistance and increase the quake. The static load test results were used to determine a relationship between ultimate resistance and quake. It was found that for a load increment of nine tons there always was an initial reading showing a settlement of 0.31 inch. Therefore, from the load tests the ratio of R_u/Q was 580,000 pounds per inch.

In Figure 3.17, the resulting curve for a quake of 0.4 inch and an ultimate resistance of 200,000 pounds is shown (R_u/Q - 500,000 pounds per inch). It is seen that spring 2, which represents the stiffness of element 3 that contains the point at which the electronic measurements were made, has zero strain when the ram rebounds off the pile. This is because the spring between the first mass and the pile cap can't carry tension. The strain in spring 3 is also plotted in this figure because it is the nearest spring to the element containing the point at which the electronic measurements were made that has the same properties of the point where the field measurements were taken (can carry tension, etc.). It is seen that the strain response closely follows the curve found by electronic measurements in the region where they once diverged. The set obtained (set - 0.582 inch per blow) is not unreasonable. It should also be noted that the peak acceleration and velocity due to initial impact are still higher for the computer solution than the electronically measured response (Figures 3.18 and 3.19).

From this study made of parameter influence on the strain response, and the response character obtained from the solution based on Smith's analysis compared to the electronically measured response, it can be concluded that the computer program can be used to simulate pile behavior.

The magnitude of the resistance offered by the ground on the pile changes the shape of the response. For the different case studied, which had different resistance characteristics, the first part of the response curves were similar. They become different when the value of the resistance becomes large enough to influence the acceleration of the segments.

Decreasing the damping along the side and at the point in direct proportion to one another, while maintaining all other parameters, results in a decrease in strain on the pile element in the region following the response due to initial impact.

Increasing the percentage of side resistance resulted in a strain response curve for the pile segment which had no sharp increases or decreases in strain shortly after the initial impact. The strain remained in a relatively small high range compared to the distribution having a lower percentage of side resistance.

Decreasing the damping factors for the side and point in direction proportion and maintaining the quake increases the set for the same ultimate resistance.

Decreasing the damping along the side and point of the pile in direction proportion causes a decrease in the rate at which the velocity of a point on the pile decreases after the initial impact.

Additional cases as well as different types of piles must be examined to be certain that these conclusions are true generally and not for the specific problem considered.

CHAPTER 4

ELEMENTARY THEORETICAL MODELS OF AN ELASTIC PILE

Although elaborate and much more accurate models of pile driving already exist (and some are discussed in Chapter 3 of this Volume), it is very useful to construct a simplified theoretical model which has at least qualitative value. Such a model aids in visualizing and assessing the pile driving effects to be seen in more complicated models or in the field.

4.1 Equation of Motion. First Model

Consider Fig. 4.1. The pile is assumed to be a uniform bar of length L and constant cross-section area A. The ground resistance will, intentionally in this first model, be drastically oversimplified to a single force at the lower tip of the pile (point bearing only.) The driving force will be considered a single function of time applied at the top of the pile. These forces will be designated by $F_1(t)$ at the top and $F_2(t)$ at the bottom, as indicated in the figure.

Let x be the distance measured along the undisturbed or rigid pile. Let ξ be the elastic displacement of the point x away from its undisturbed position. Let X be the overall displacement of the x coordinate system; X is thus like a rigid body displacement of the whole pile.

We next derive the equation for the force equilibrium of an element of the pile; see Figure 4.2.

An element of length dx will be acted upon by a positive (tensile) stress σ at x and a positive tensile stress of $\sigma + \frac{\partial \sigma}{\partial x}$ dx at x + dx. The corresponding forces are obtained by assuming these stresses to be uniformly distributed over the section area A. The strain at x is $\varepsilon = \frac{\partial \xi}{\partial x}$, so that assuming elastic material of the pile with a modulus of elasticity of E:

$$\sigma = E \frac{\partial \xi}{\partial x} \qquad 4.1$$

If ρ is the density of the pile material, then equating elastic and gravitational to inertial forces in the x-direction leads to the equation

$$A \frac{\partial \sigma}{\partial x} dx + \rho A g \, dx = \rho A dx \frac{\partial^2}{\partial t^2} (X + \xi) \qquad 4.2$$

In view of the elastic relation of stress to strain this simplifies to

$$EA \frac{\partial^2 \xi}{\partial x^2} + \rho A g = \rho A (\ddot{X} + \ddot{\xi}) \qquad 4.3$$

or, if $c = \sqrt{E/\rho}$ is the speed of sound in the pile:

$$\frac{\partial^2 \xi}{\partial x^2} - \frac{1}{c^2} \ddot{\xi} = \frac{1}{c^2} (\ddot{X} - g) \qquad 4.4$$

This is the wave equation of the pile. It is subject to the boundary conditions

$$\left.\frac{\partial \xi}{\partial x}\right|_{x=0} = \frac{-F_1(t)}{EA} \; ; \quad \left.\frac{\partial \xi}{\partial x}\right|_{x=L} = \frac{-F_2(t-L/_c)}{EA} \qquad 4.5$$

which state that the strain is compressive under the forces F_1 and F_2 at top and bottom, respectively. The assumption is made throughout that stresses are uniformly distributed over any cross-sectional area.

4.2 General Solutions of the Equation of Motion, First Model

To obtain a solution to equation 4.4 under conditions 4.5 it will be assumed that the pile initially has no displacements or velocity (quiescent). Taking Laplace transforms ($L\ \xi = \bar{\xi}$, etc.) of the quantities in equations 4.4 and 4.5 yields

$$\frac{d^2\bar{\xi}}{dx^2} - \frac{p^2}{c^2}\bar{\xi} = \frac{1}{c^2}\left(p^2\,\bar{X} - \frac{g}{p}\right) \qquad 4.6$$

$$\left.\frac{d\bar{\xi}}{dx}\right|_{x=0} = \frac{-\bar{F}_1(t)}{EA}$$

$$\left.\frac{d\bar{\xi}}{dx}\right|_{x=L} = \frac{-\bar{F}_2(t-4_c)}{EA} \qquad 4.7$$

Since $\bar{X}$ is a function of time only, and not of x, the solution of equations 4.6 and 4.7 is

$$\bar{\xi} = \frac{c}{p \sinh \frac{pL}{c}}\left[\frac{\bar{F}_1}{EA}\cosh\frac{pL}{c} - \frac{\bar{F}_2}{EA}\right]\cosh\frac{px}{c}$$

$$\frac{-c}{p}\ \frac{\bar{F}_1}{EA}\sinh\frac{px}{c} - \bar{X} + \frac{g}{p^3} \qquad 4.8$$

Since the top (x=0) of the pile will be singled out for examination (this is the point where, in the field, strain and acceleration measurements are made), the value of x in equation 4.8 can be set equal to zero. Thus the transform of the pile-top displacement $\xi + X$ is

$$(\bar{\xi} + \bar{X})\Big|_{x=0} = \frac{c}{p \sinh \frac{pL}{c}} \left[\frac{\bar{F}_1}{EA} \cosh \frac{pL}{c} - \frac{\bar{F}_2}{EA}\right] + \frac{g}{p^3} \qquad 4.9$$

The inversion of equation 4.9 takes several forms. First are stated four alternative general solutions:

$$(X + \xi)_{x=0} = \frac{c}{EA} \int_0^t H(t-\tau)F_1 \ (\tau)\, d\tau + gt^{2/2}$$

$$+ \frac{2c}{EA} \int_0^t [H(t-\tau- \frac{2L}{c}) + H(t-\tau- \frac{4L}{c}) + \ldots]\, F_1(t-\tau)\, d\tau$$

$$- \frac{2c}{EA} \int_0^t [H(t-\tau- \frac{L}{c}) + H(t-\tau- \frac{3L}{c}) + \ldots]\, F_2(\tau)\, d\tau \qquad 4.10a$$

$$(X + \xi)\Big|_{x=0} = \frac{c}{EA} \int_0^t [H(\tau)\ F\ (t-\tau)\, d\tau + gt^{2/2}$$

$$+ \frac{2c}{EA} \int_0^t [H(\tau- \frac{2L}{c}) + H(\tau-\frac{4L}{c}) + \ldots]\, F_1(t-\tau)\, d\tau$$

$$- \frac{2c}{EA} \int_0^t [H(\tau- \frac{L}{c}) + H(\tau- \frac{2L}{c}) + \ldots]\, F_2(t-\tau)\, d\tau \qquad 4.10b$$

$$(X + \xi)\Big|_{x=0} = \int_0^t F_1(\tau)\ x'_{R1}(t-\tau)\, d\tau + \int_0^t F\ (\tau)\ x'_{R2}(t-\tau)\, d\tau$$

$$+ \frac{gt^2}{c} \qquad 4.10c$$

$$(X + \xi)\Big|_{x=0} = \int_0^t F_1'(\tau)\, x_{R1}\,(t-\tau)\, d\tau + \int_0^t F_2'(\tau)\, x_{R2}(t-\tau)\, d\tau$$

$$+ gt^2/2 \qquad 4.10d$$

In the above solutions $H(t-\tau)$ represents the Heaviside unit step function rising from zero to unit value at $t = \tau$; x_{R1} and x_{R2} are the solutions of equation 4.9 when respectively $F_1 = H(t)$ $F_2 = 0$ or $F_1 = 0$, $F_2 = H(t)$, i.e. the responses of the pile to unit step inputs at either end; primes indicate first derivatives with respect to the arguments indicated.

To render equations (4.10c) and (4.10d) more explicit, consider the case $F_1 = F_{10}\, H(t)$, $F_2 = 0$. In this case $\overline{F}_1 =$ F_{10}/p and equation (4.9) becomes

$$(\overline{\xi} + \overline{X})\Big|_{x=0} = \frac{c}{p^2 \sinh \frac{pL}{c}}\left[\frac{F_{10}}{EA} \cosh \frac{pL}{c}\right] + \frac{g}{p^3} \qquad 4.11$$

But

$$\frac{\cosh \frac{pL}{c}}{\sinh \frac{pL}{c}} = 1 + 2\left[e^{-\frac{2pL}{c}} + e^{-\frac{4pL}{c}} + e^{-\frac{6pL}{c}} + \ldots\right] \qquad 4.12$$

Hence, multiplying by p^2:

$$p^2(\overline{\xi} + \overline{X})\Big|_{x=0} = L(\ddot{\xi} + \ddot{X})\Big|_{x=0} = \frac{F_{10}c}{EA}\left[1 + 2\left[e^{-\frac{2pL}{c}} + e^{-\frac{4pL}{c}} + \ldots\right]\right] + \frac{g}{p} \qquad 4.13$$

* Note that x_{R1}' and x_{R2}' are also respectively the responses to δ-function inputs (impulses) at top and bottom.

Inverting yields the acceleration:

$$(\ddot{\xi} + \ddot{X})\Big|_{x=0} = \frac{F_{10}c}{EA}\left[\delta(t) + 2\left[\delta(t-\frac{2L}{c}) + \delta(t-\frac{4L}{c}) + \ldots\right]\right] + g \qquad 4.14$$

where $\delta(t - \tau)$ is the Dirac delta function (or "infinite spike" at $t = \tau$).

The first integral of this expresses the velocity:

$$(\dot{\xi} + \dot{X})\Big|_{x=0} = \frac{F_{10}c}{EA}\left[H(t) + 2\left[H(t-\frac{2L}{c}) + H(t-\frac{4L}{c}) + \ldots\right]\right] + gt \qquad 4.15$$

and a final integration yields the displacement:

$$F_{10}x_{R1} = (\xi + X)\Big|_{x=0} = \frac{F_{10}c}{EA}\left[t + 2\left[H(t-\frac{2L}{c})(t-\frac{2L}{c}) + H(t-\frac{4L}{c})(t-\frac{4L}{c}) + \ldots\right]\right] + gt^2/2 \qquad 4.16$$

This exhibits the explicit form which x_{R1} takes on for use in equations (4.10c) or (4.10d); x_{R2} may be obtained analogously.

It is interesting for illustrative purposes to select one of the simple solutions of equation 4.14, 4.15 or 4.16 and to compare it with the corresponding result obtained with the assumption that the pile is a rigid body (incapable of ξ-motion). To this end, 4.15 is selected and compared with results of the following brief rigid-body analysis.

Let m be the mass ρAL of the pile. Then, under a unit step force $F_1(t) = F_{10}\,H(t)$:

$$m \ddot{X} = F_1 (t) + m g \qquad 4.17a$$

or

$$\ddot{X} = \frac{F_{10}}{m} H(t) + g \qquad 4.17b$$

Integration yields:

$$\dot{X} = \frac{F_{10}}{m} t + gt \qquad 4.18$$

The results of equations 4.15 and 4.18 for velocity at the top of the pile are rendered non-dimensional by dividing velocity, V, by the factor $\frac{F_{10}}{pAc}$; they are then plotted as shown in Figure 4.3, where gravity effects are neglected.

An interpretation of the physical significance of Figure 4.3 lends insight into the mechanism of pile response. Under the sudden application of a constant force at the top, the rigid pile, jumping to a finite acceleration, increases velocity at a linear rate. On the other hand, the elastic pile responds in steps separated by the travel time 2L/c of a wave down the length L and back at sound velocity c. The "average" of the elastic velocity response, however, is seen to be the rigid body velocity response.

Figure 4.3 serves as a basic form from which a number of solutions can be constructed. This fact is implicit in equation 4.10c or 4.10d since a general variety of functions F(t) can be constructed from H(t), but simple qualitative reasoning using Figure 4.3 suffices to get a number of instructive results very easily.

Suppose for example that $F_1(t)$ consists of a square pulse of duration T and height F_{10}, i.e. in the language used above:

$$F_1(t) = [H(t) - H(t-T)]\ F_{10} \qquad 4.19$$

To obtain the velocities then it suffices to subtract, from the given results, results similar to those of Figure 4.3, starting at t = T. Taking for illustrative purposes. T = L/c yields Figure 4.4

In this figure the rigid body and elastic body results are seen to be rather dissimilar, but still the rigid body results are a kind of "average" of the elastic body results. The short force pulse at the top, of duration T, is seen to traverse the pile and reflect back to the top in intervals of $\frac{2L}{c}$, the time required for wave passage at sonic speed. It is to be emphasized that in the actual case, successive reflections would probably be damped out, so that the figure represents only a hypothetical situation.

If, referring again to Figure 4.4, we replace the single top force pulse by a half sine wave pulse of the same amplitude and duration (taking $F_2 \equiv 0$), the results are as shown in Figure 4.5.

Further cases can be developed, for example assuming various rules of action governing the force $F_2(t)$ at the lower tip. One possible situation is discussed here. Since, in fact, the force $F_2(t)$ is a soil reaction force, it would be reasonable to assume that this force does not arise until t = L/c, i.e. until the top stress wave hits the bottom. Assuming for example that

$$F_1\ (t) = F_{10}\ \delta\ (t) \qquad \left(\int_0^\infty \delta(t)\ dt = 1 \right) \qquad 4.20$$

$$F_2\ (t) = F_{20}\ \ \delta(t - \frac{L}{c})$$

i.e. that the two forces are hammer blows of "infinite height and infinitesmal duration" (in fact that they are merely devices for transfer of units of mementum) the result for the transform of $\xi + X$ at $x = 0$ from equation 4.9 is

$$(\bar{\xi} + \bar{X})\Big|_{x=0} = \frac{c}{EAp} \Bigg[[1 + 2\ (e^{-\frac{2pL}{c}} + e^{-\frac{4pL}{c}} + e^{-\frac{6pL}{c}} + \ldots)]\ F_{10}$$

$$- 2[e^{-\frac{2pL}{c}} + e^{-\frac{4pL}{c}} + e^{-\frac{6pL}{c}} + \ldots]\ F_{20} \Bigg] + g/p^3 \qquad 4.21$$

of for the transform of the velocity:

$$p(\bar{\dot{\xi}}\ \ \bar{\dot{X}})\Big|_{x=0} = \frac{c}{EA} \Bigg[F_{10} + 2(F_{10} - F_{20})(e^{-\frac{2pL}{c}} + e^{-\frac{4pL}{c}} + e^{-\frac{6pL}{c}}$$

$$+ \ldots) \Bigg] + g/p^2 \qquad 4.22$$

This yields for the velocity:

$$(\dot{\xi} + \dot{X})\Big|_{x=0} = \frac{cF_{10}}{EA}\ \delta(t) + \frac{2c}{EA}\ (F_{10} - F_{20})[\delta(t - \frac{2L}{c}) + \delta\ (t - \frac{4L}{c})$$

$$+ \ldots] + gt \qquad 4.23$$

and for the displacement:

$$(\xi + X)\Big|_{x=0} = \frac{cF_{10}}{EA} \quad H(t) + \frac{2c}{EA}\ (F_{10} - F_{20})\ [H(t - \frac{2L}{c}) + H(t - \frac{4L}{c}$$

$$+ \ldots] + gt^2/2 \qquad 4.24$$

This can be conveniently plotted (taking $2F_{20} = F_{10}$ arbitrarily) if $\frac{F_{10}}{\rho Ac}$ is used to reduce the amplitude to a dimensionless quantity. The result is given Figure 4.6. The rigid body result

$$X - \frac{gt^2}{2} = \frac{F_{10}}{m} H(t)\ t - \frac{F_{20}}{m} [H(t-\frac{L}{c})\ (t-\frac{L}{c})] \qquad 4.25$$

is also indicated in this figure.

4.3 Equation of Motion. Second Model

Because of the relatively poor simulation of reality in the first model, a second one is introduced in Figure 4.7. In this case, the soil resistance will be simulated by a uniformly distributed force f_r(lb./ft.) along the pile, where

$$f_r = K_1 + K_2\ (\dot{X} + \dot{\xi}) + K_3\ (\dot{X} + \dot{\xi})^2 + \dots \qquad 4.26$$

i.e. f_r may consist of a frictional force K_1, a viscous force $K_2(\dot{X}+\dot{\xi})$ and higher order resistive forces in powers of the absolute pile velocity. Actually, in what follows, only the K_1 and K_2 terms will be retained.

Again assume a uniform pile of cross section, A, which receives an impulsive blow $F_o\ \delta(t)$ on top (Figure 4.7).

$$\rho A\ dx\ (\ddot{X} + \ddot{\xi}) = A\ \frac{\partial \sigma}{\partial x} dx - f_r\ dx \qquad 4.27$$

This leads to the following equation of motion:

$$\frac{\partial^2 \xi}{\partial x^2} - \frac{1}{c^2} (\ddot{X} + \ddot{\xi}) - \beta(\dot{X} + \dot{\xi}) = \alpha \qquad 4.28$$

where $c^2 = E/\rho$ and

$$\alpha = \frac{K_1}{AE} \qquad 4.29$$

$$\beta = \frac{K_2}{AE}$$

Quiescent initial conditions are assumed:

$$X + \xi = 0 \qquad \dot{X} + \dot{\xi} = 0 \qquad (t = 0) \qquad 4.30$$

Taking Laplace transforms of equation 4.28 yields

$$\frac{d^2\bar{\xi}}{dx^2} - \lambda^2 \bar{\xi} = \frac{\alpha}{p} + \lambda^2 \bar{X} \qquad 4.31$$

where

$$\lambda^2 = \frac{p^2}{c^2} + \beta p$$

The general solution of equation 4.31 is

$$\bar{\xi} = A_1 \cosh \lambda x + B_1 \sinh \lambda x - \frac{\alpha}{\lambda^2 p} - \bar{X} \qquad 4.32$$

The boundary conditions on the pile are

$$\frac{\partial \xi}{\partial x} = \frac{-F_o \delta(t)}{AE} \quad \text{at } x = 0$$

$$\partial\xi/\partial x = 0 \quad \text{at } x = L \qquad 4.33$$

which transform to

$$\left.\frac{\partial \bar{\xi}}{\partial X}\right|_{X=0} = \frac{-F_o}{AE} \; ; \quad \left.\frac{\partial \bar{\xi}}{\partial X}\right|_{X=L} = 0 \qquad 4.34$$

Using conditions 4.34 in equation 4.32 yields the solution

$$\bar{X} + \bar{\xi} = \frac{F_o}{AE\lambda} \frac{\cosh \lambda L}{\sinh \lambda L} \cosh \lambda \chi - \frac{F_o}{AE\lambda} \sinh \lambda \chi - \frac{\alpha}{\lambda^2 p} \qquad 4.35$$

4.4 Solution to Equation of Motion; Second Model

Before inverting equation 4.35 we concentrate attention on the top of the pile x = 0. At this point

$$\bar{X} + \bar{\xi} = \frac{F_o}{AE} \frac{\cosh \lambda L}{\lambda \sinh \lambda L} - \frac{\alpha}{\lambda^2 p} \qquad 4.36$$

The inversion of equation 4.36, after using a series expansion for the term containing hyperbolic functions, yields:

$$X + \xi = \frac{F_o c}{AE} e^{-\frac{\beta c^2}{2} t} \Big[I_o\left(\frac{\beta c^2}{2} t\right) + 2 \Big[I_o \left(\frac{\beta c^2}{2} \sqrt{t^2 - \left(\frac{2L}{c}\right)^2}\right)$$

$$+ I_o \left(\frac{\beta c^2}{2} \sqrt{t^2 - \left(\frac{4L}{c}\right)^2}\right) + I_o \left(\frac{\beta c^2}{2} \sqrt{t^2 - \left(\frac{6L}{c}\right)^2}\right) + \dots \Big] \Big]$$

$$+ \frac{K_1}{K_2} \Big[\frac{1}{\beta c^2} - t - \frac{1}{\beta c^2} e^{-\beta c^2 t} \Big] \qquad 4.37$$

where I_o is the hyperbolic Bessel function having the property that $I_o \left(\frac{\beta c^2}{2} \sqrt{c^2 - \tau^2}\right) = 0$ for $t < \tau$. The derivative of I_o is $\frac{dI_o}{dt} = I_1$ where these functions have the character suggested in Figure 4.8 and are expressed by the following formulas:

$$I_o(Z) = 1 + \frac{Z^2}{4} + \frac{Z^4}{64} + \frac{Z^6}{2304} + \dots$$

$$I_1(Z) = \frac{Z}{2} + \frac{Z^3}{16} + \frac{Z^5}{384} + \dots$$

The pile top velocity is obtained by differentiating equation 4.37:

$$\mathring{X} + \mathring{\xi} = \frac{F_o \beta c^3}{2AE} e^{-\frac{1}{2}\beta c^2 t} \left\{ -[I_o(\frac{\beta c^2}{2}t) + 2(I_o(\frac{\beta c^2}{2}\sqrt{t^2-(\frac{2L}{c})^2}) \right.$$

$$+ I_o(\frac{\beta c^2}{2}\sqrt{t^2-(\frac{4L}{c})^2}) + I_o(\frac{\beta c^2}{2}\sqrt{t^2-(\frac{6L}{c})^2}) + \ldots)] + [I_1(\frac{\beta c^2}{2}t)$$

$$+ 2(\frac{t}{\sqrt{t^2-(\frac{2L}{c})^2}} I_1(\frac{\beta c^2}{2}\sqrt{t^2-(\frac{2L}{c})^2}) + \frac{t}{\sqrt{t^2-(\frac{4L}{c})^2}} I_1(\frac{\beta c^2}{2}) X$$

$$\left. (\sqrt{t^2-(\frac{4L}{c})^2}) + \ldots)] \right\} + \frac{K_1}{K_2}(e^{-\beta c^2 t} - 1) \qquad 4.38$$

Let $\rho AL = m$ be the mass of the rigid pile. The equation of motion is

$$m \mathring{X} + K_2 L \dot{x} = F_o \delta(t) - K_1 L \qquad 4.39$$

with quiescent initial conditions. This is equivalent to the equation

$$m \ddot{X} + K_2 L \mathring{X} + K_1 L = 0 \qquad 4.40$$

with the initial conditions:

$$x(0) = 0 \qquad \mathring{X}(0) = F_o/m \qquad 4.41$$

Let

$$X = A + Bt + Ce^{pt} \qquad 4.42$$

be a trial solution of equation 4.40 for conditions 4.41. Use of this results in the solution

$$X = \frac{m}{K_2 L}\left[\frac{F_o}{m} + \frac{K_1}{K_2}\right] - \frac{K_1}{K_2}t - \frac{m}{K_2 L}\left[\frac{F_o}{m} + \frac{K_1}{K_2}\right] e^{-\frac{LK_2 t}{m}} \qquad 4.43$$

for which the velocity is

$$\dot{X} = \frac{-K_1}{K_2} + [\frac{F_o}{m} + \frac{K_1}{K_2}] e^{-\frac{K_2 Lt}{m}} \qquad 4.44$$

Taking $K_1 = 0$ and $\beta = \frac{1}{LC}$ (i.e. $K_2 = \beta AE = \frac{AE}{LC}$) permits a sample plot of responses as given in equations 4.37 and 4.43, 4.38 and 4.44 and shown in Figures 4.9 and 4.10. Other plots can be carried out similarly. Note that the velocity at the top of the elastic pile goes through infinite positive jumps at multiples of $\frac{2L}{C}$ with negative stretches between jumps.

If $y_p(t)$ be any response of the system (such as displacement, velocity, acceleration) to an impulse, such as is applied in the second model, and if F(t) to be the actual force applied to the pile, then the total corresponding response Y(t) is

$$Y(t) = \int_0^t F(\tau)\, y_p\, (t-\tau)\, d\tau \qquad 4.45$$

The two theoretical pile models assumed above are not sufficiently realistic as yet to permit more than qualitative conclusions for actual piles. They do clearly indicate, however, the effect of wave action traveling up and down the pile, and the oscillation of the elastic pile about the rigid body position. The actual forcing functions encountered in practice are never true impulse-functions, and this fact may be expected to "spread out" the response graph for more realistic forcing functions.

It may be noted from equation 4.45 that the form of the response tends to follow the form of the forcing function F(t) with slight "deviations" imposed by the $y_p(t)$, which tends toward a unit value in either the elastic or the rigid body case. In fact, the elastic response in a sense "oscillates" about the rigid body response and converges to it in time.

The continuing aim of the present theoretical approach is to develop sufficiently realistic pile models and forcing functions for them to reveal the truly important differences - if any-between rigid body pile and elastic pile response. To date the models and loadings used have suggested methods of approach but have not settled the question of the extent the use of a rigid body pile model - as against an elastic one -, is adequate in the present study. While, theoretically, strong differences between the two models appear to exist, the practical results of field experience to date suggest that a rigid body model may in fact be quite adequate.

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TABLE 3.1

DRIVING RECORD FOR THE PILE INVESTIGATED

Penetration (feet)	Blows	Set (Inches/Blow)
50	24	0.500
51	22	0.545
52	23	0.522
53	27	0.444

Driving record for the last four feet of driving. Pile located at Willow-Clark Interchange, Cleveland, Ohio.

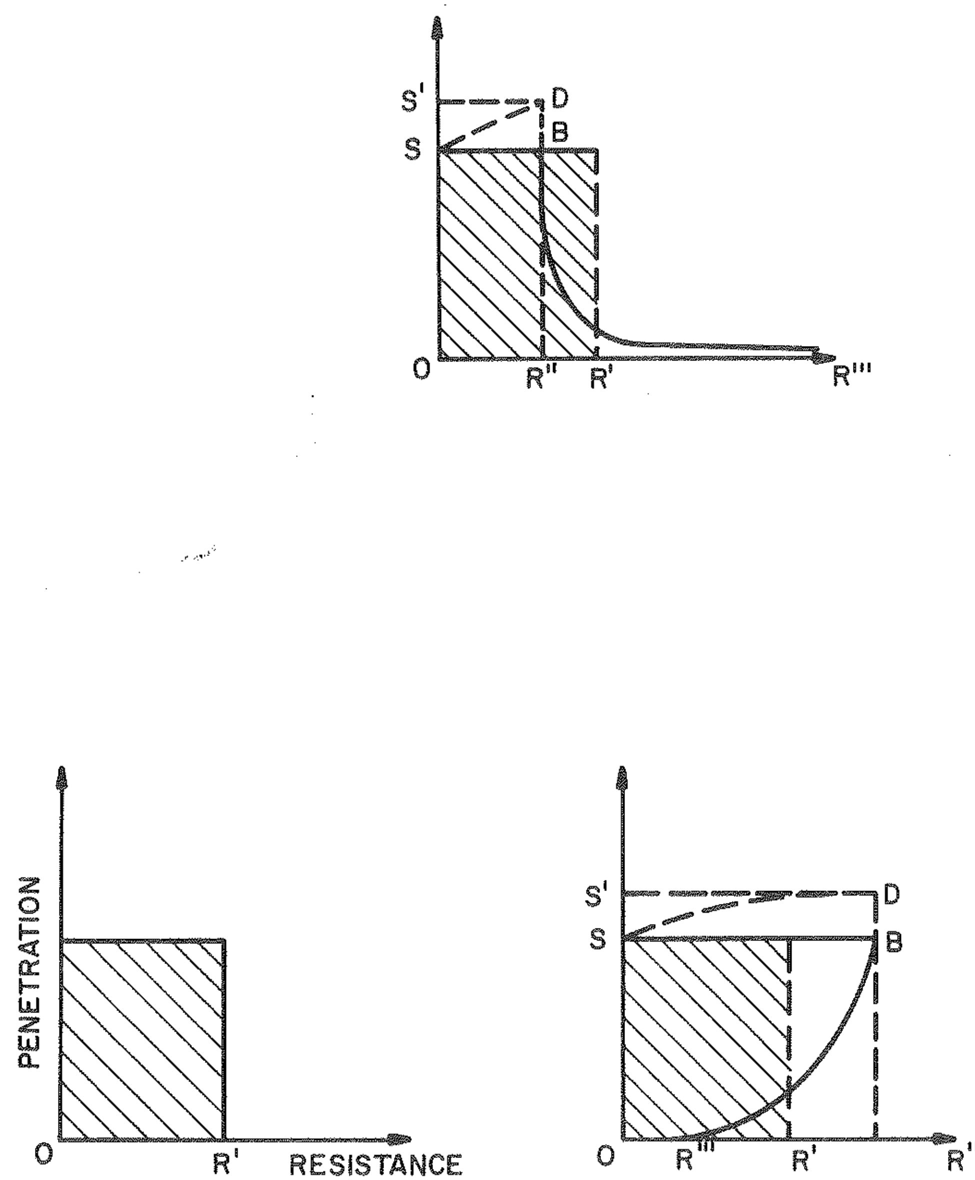


Figure 2.1

Work Diagrams

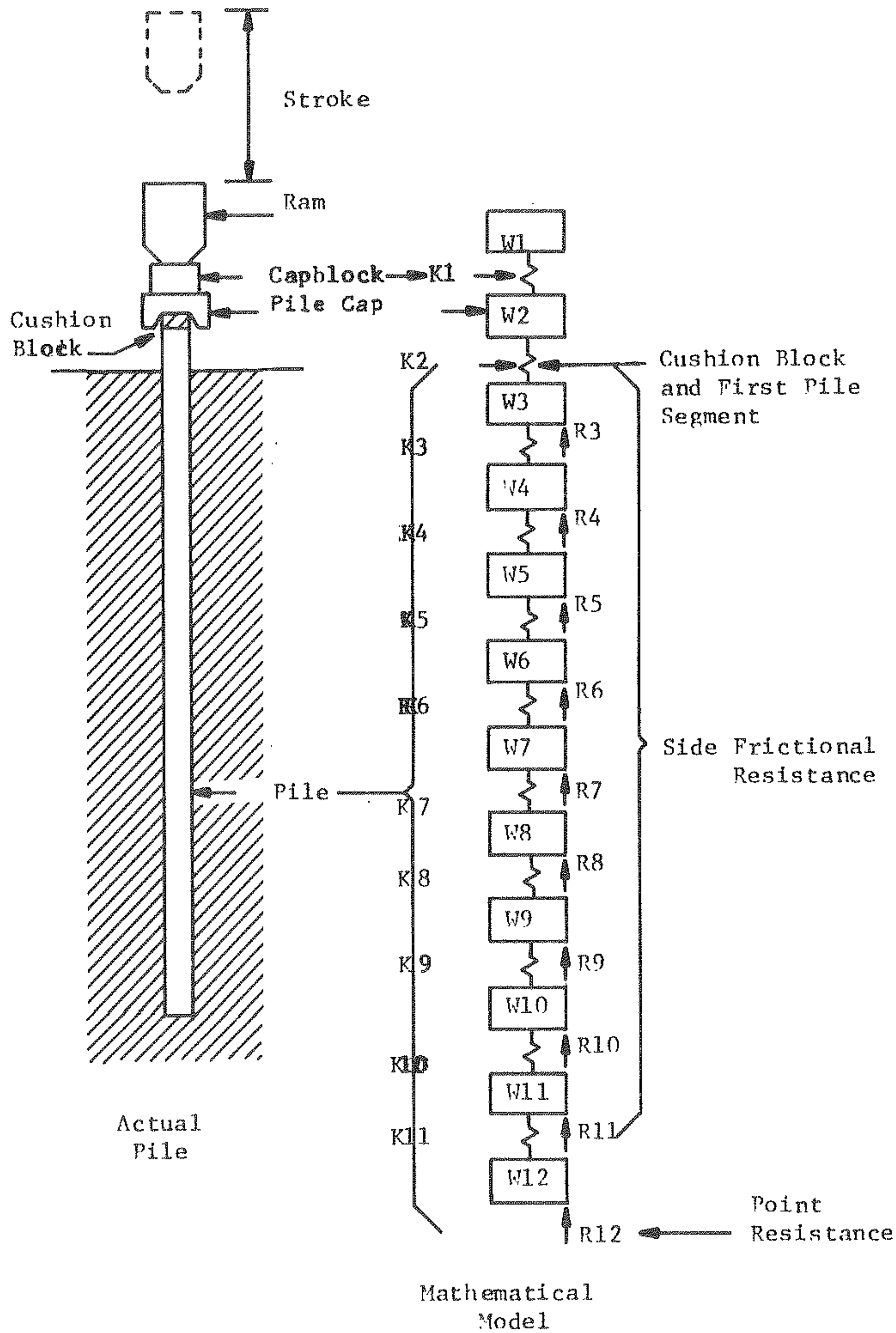
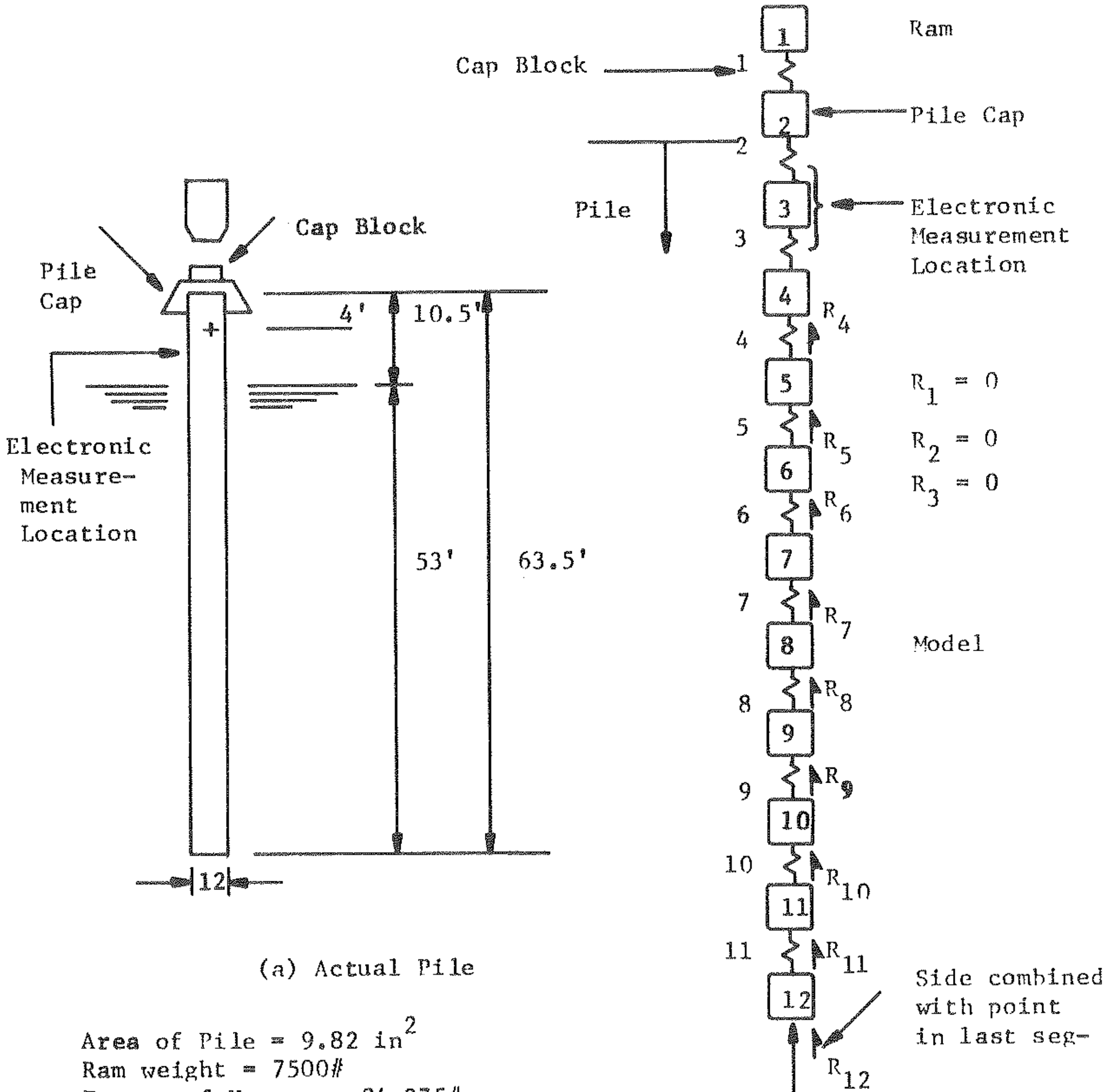


FIGURE 3.1

Mathematical Model of Typical Pile



(a) Actual Pile

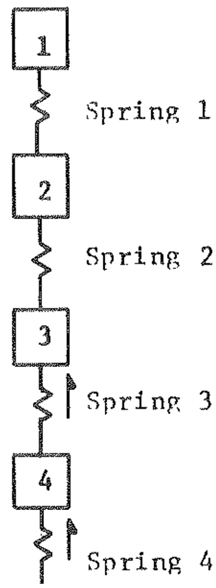
Area of Pile = 9.82 in^2
Ram weight = 7500#
Energy of Hammer = 24,375#
Steel Pile
Assumed Stiffness of Cap Block
= 2 x 10^6 #/in
Assumed weight of Pile Cap = 700#

(b) Mathematical Model

Field Test Pile, December 16, 1964
Location: Willow Freeway Over Clark Freeway
Willow - Clark Interchange

FIGURE 3.2

Mathematical Model of Pile Studied

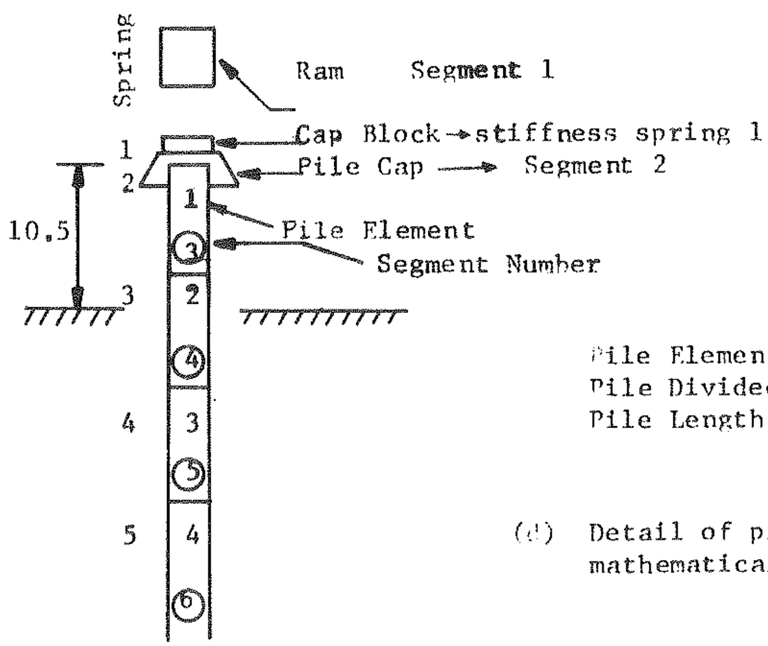


Spring 1: Carries no tension, has stiffness of cap-block.

Spring 2: Carries no tension, has stiffness of pile element 1.

Spring 3: Carries tension, has stiffness of pile element 2.

(c) Special spring characteristics.



Pile Element Lengths = 6.35'
Pile Divided into ten segments
Pile Length = 63.5'

(d) Detail of pile division for mathematical model.

FIGURE 3.2

Continued

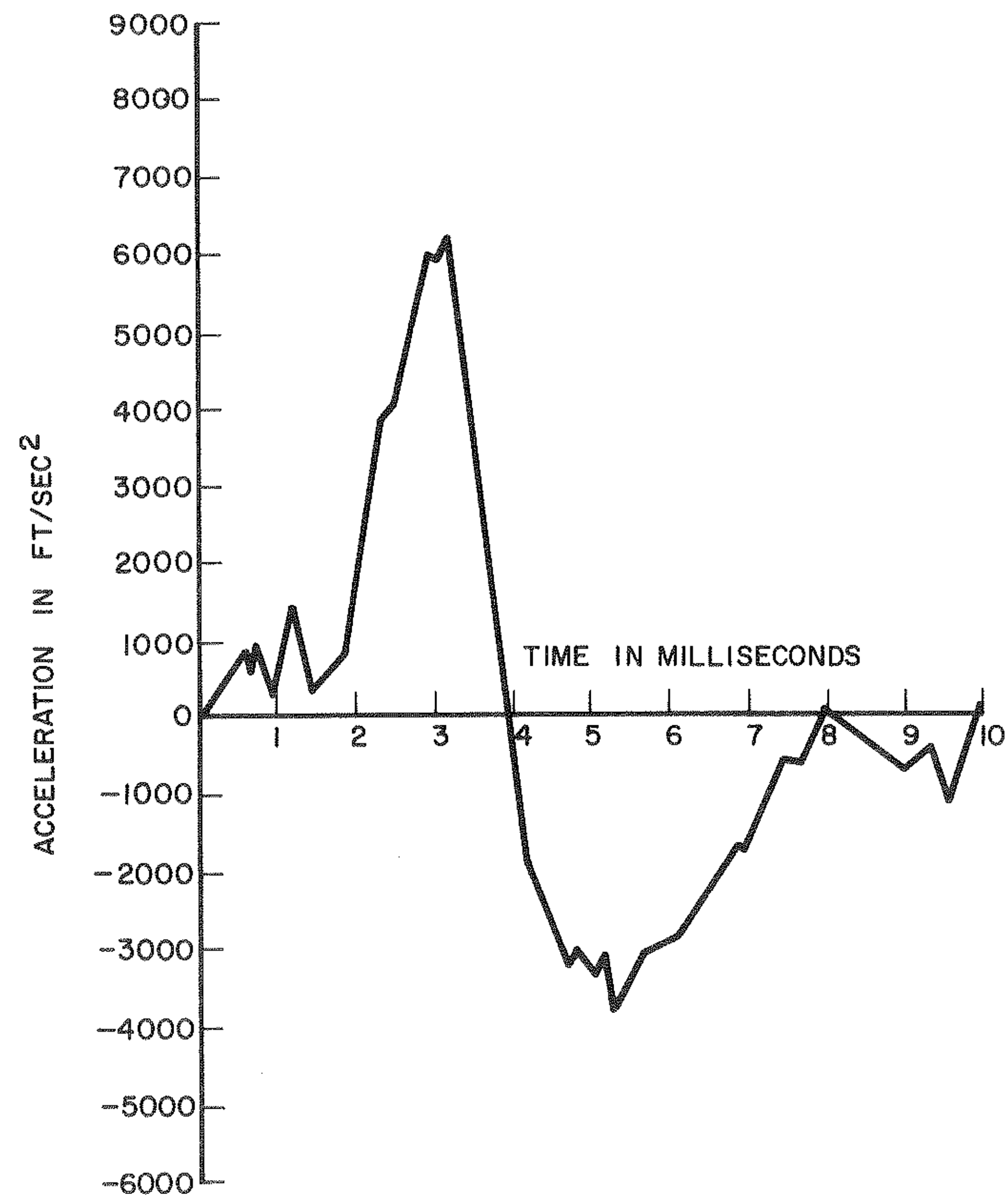


Figure 3.3

Acceleration Versus Time - Electronically Recorded

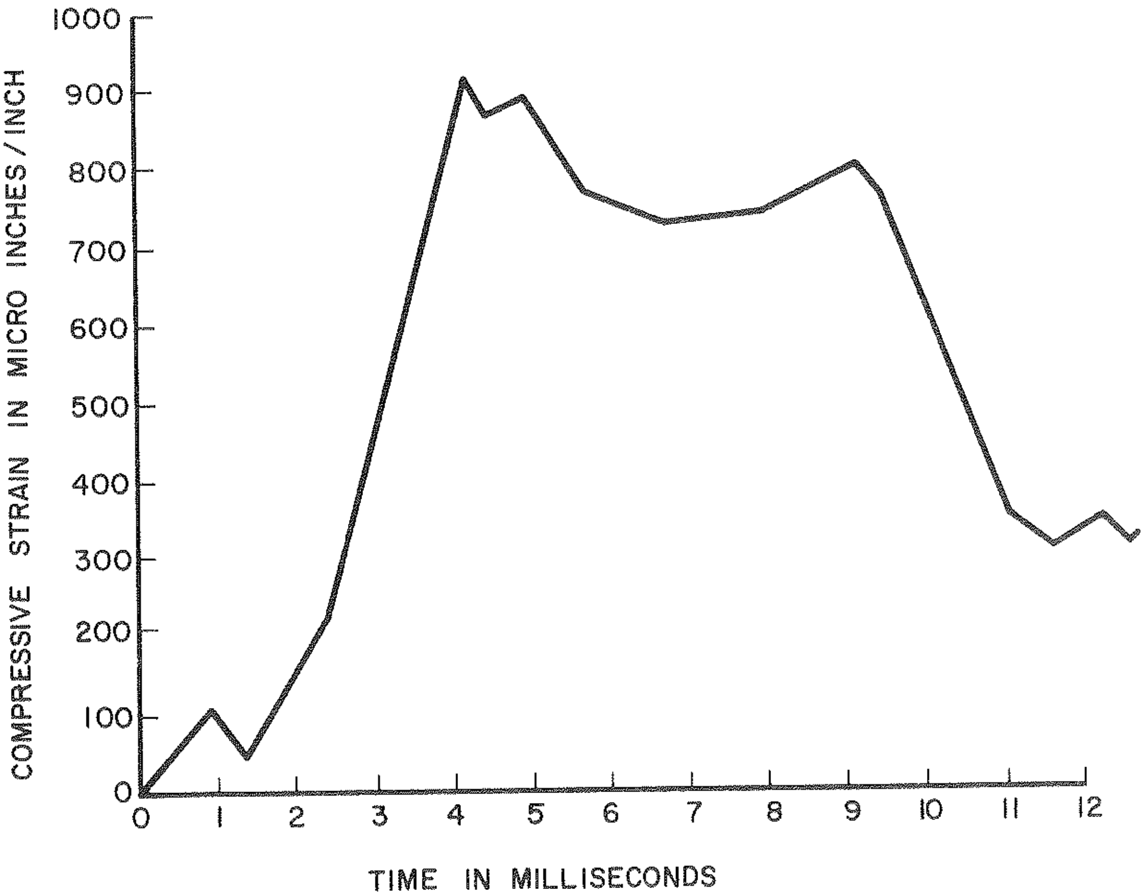


Figure 3.4

Electronically Recorded Strain

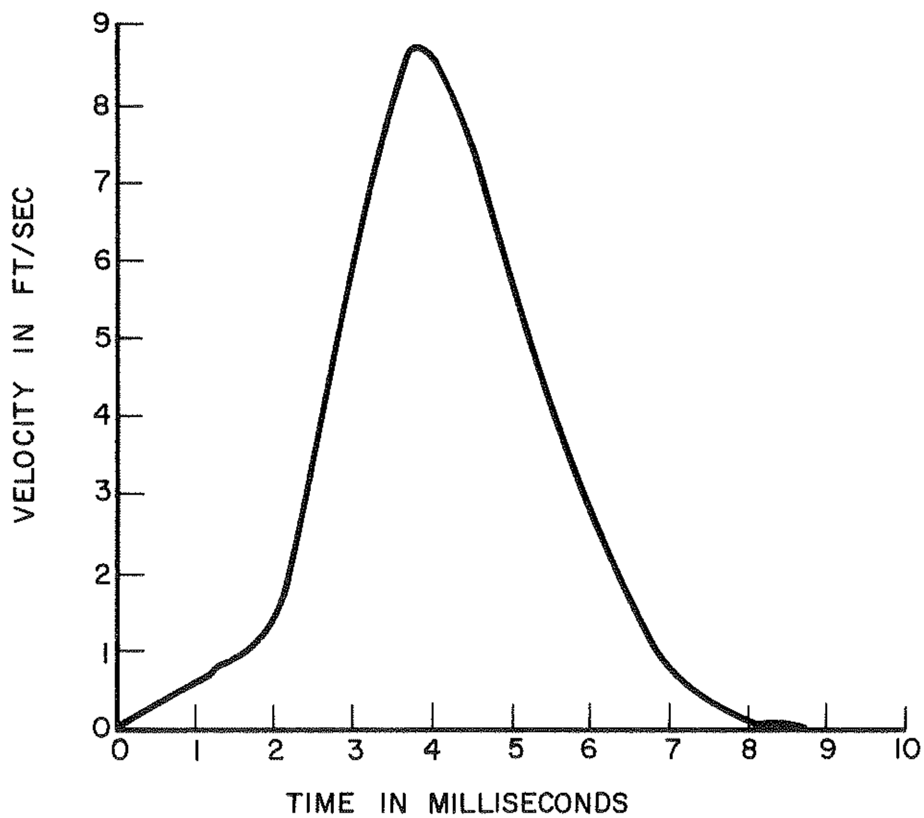


Figure 3.5

Velocity Integrated from
Electronically Recorded Acceleration

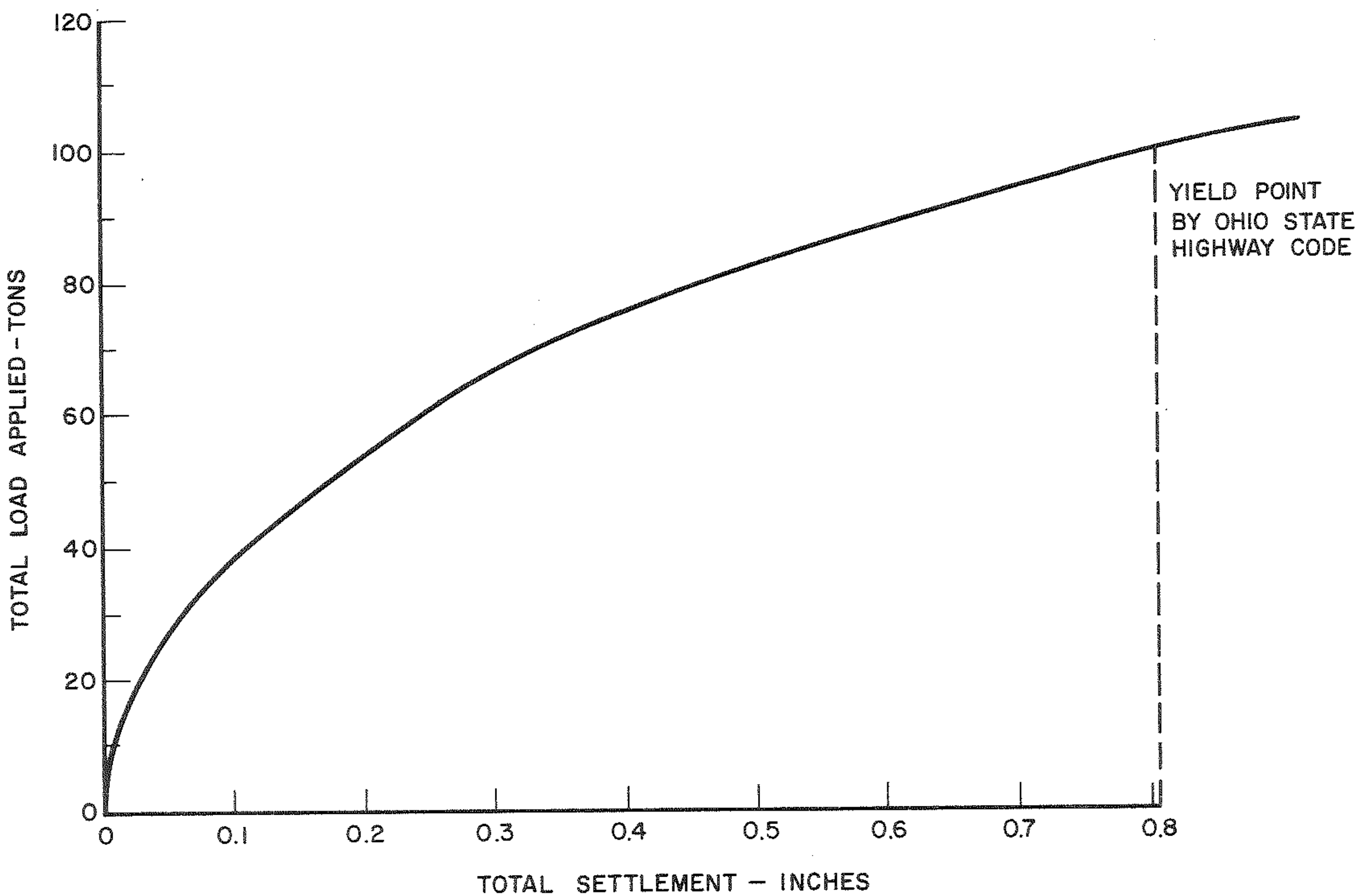


Figure 3.6

Load Settlement Curve for Pile Investigated Under Load Test

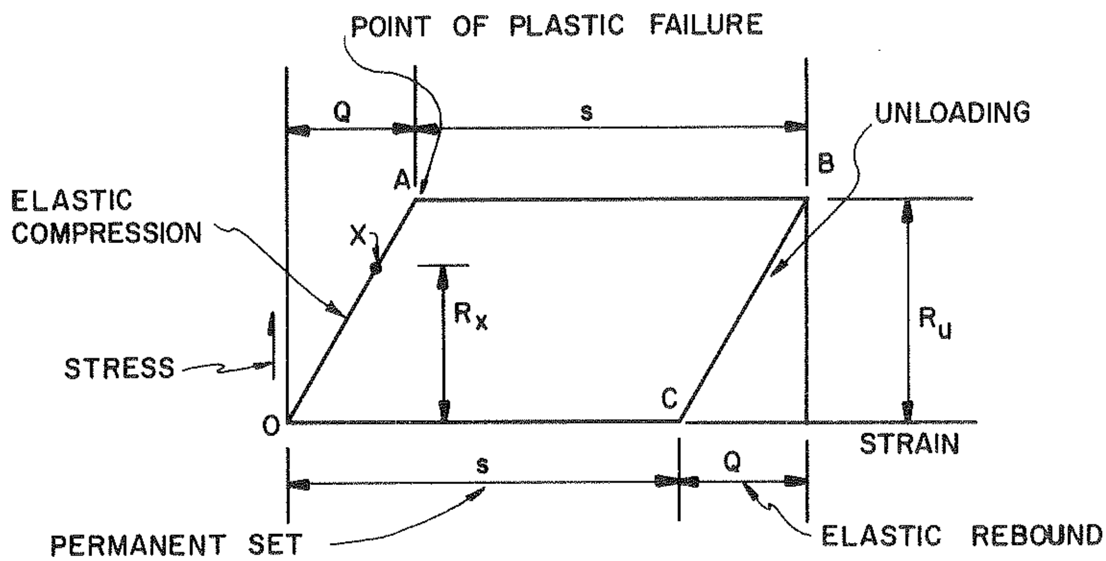
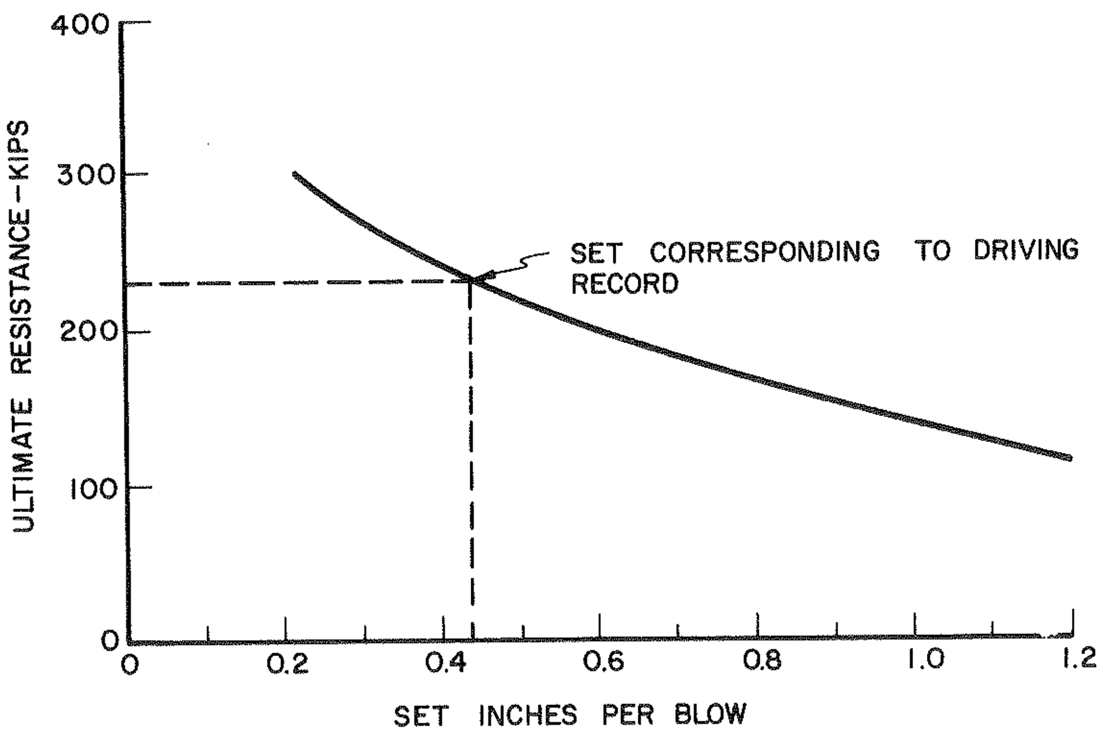


Figure 3.7

Stress-Strain Diagram at Pile Point



R_u = 230 Kips

Resistance Distribution	Point = 69% R_u Side = 31% R_u
Damping and Quake Values	J = 0.15 J' = 0.05 Q = 0.1

Set = 0.44 Inch/Blow

Figure 3.8

Set Versus Ultimate Resistance

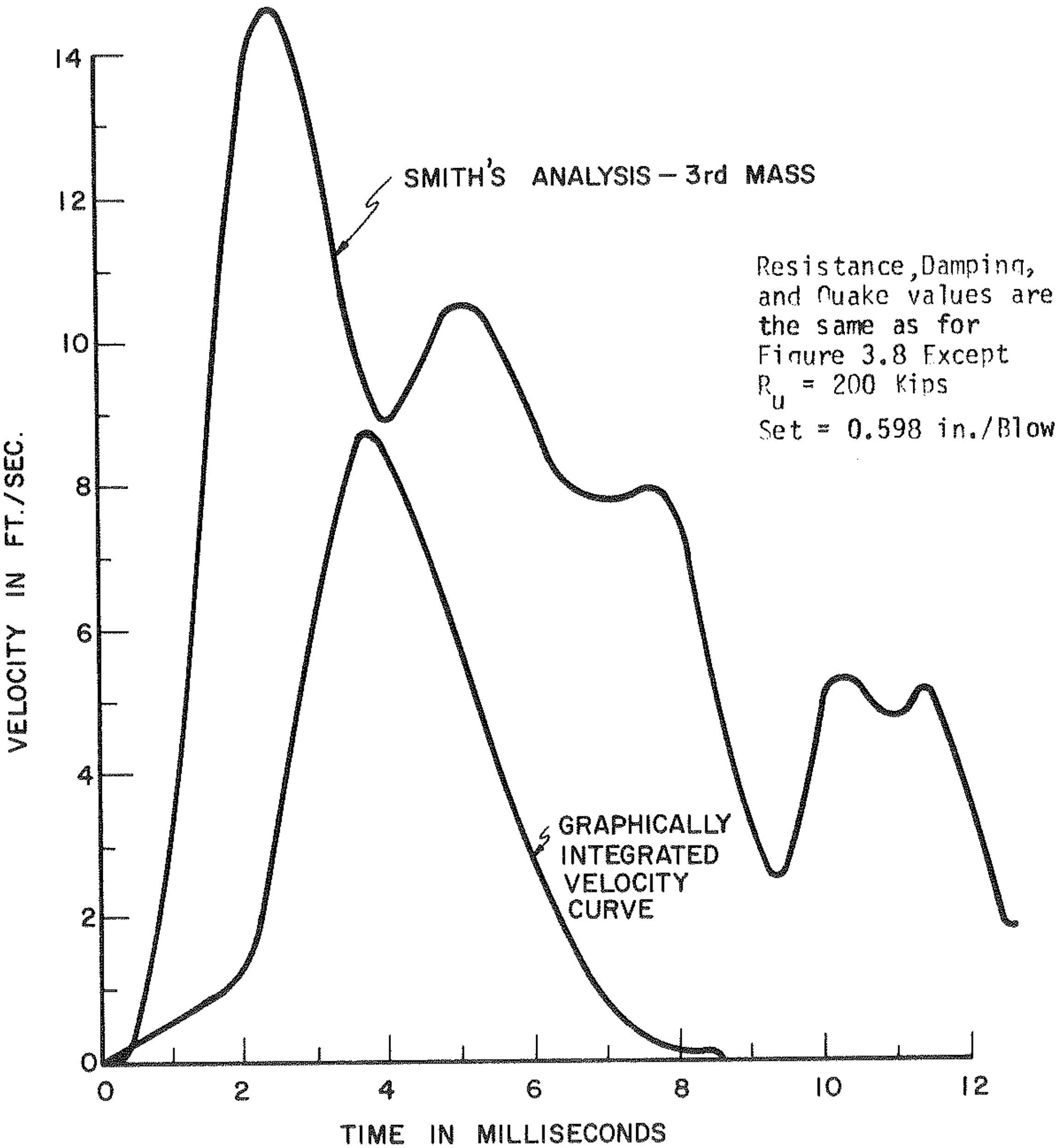
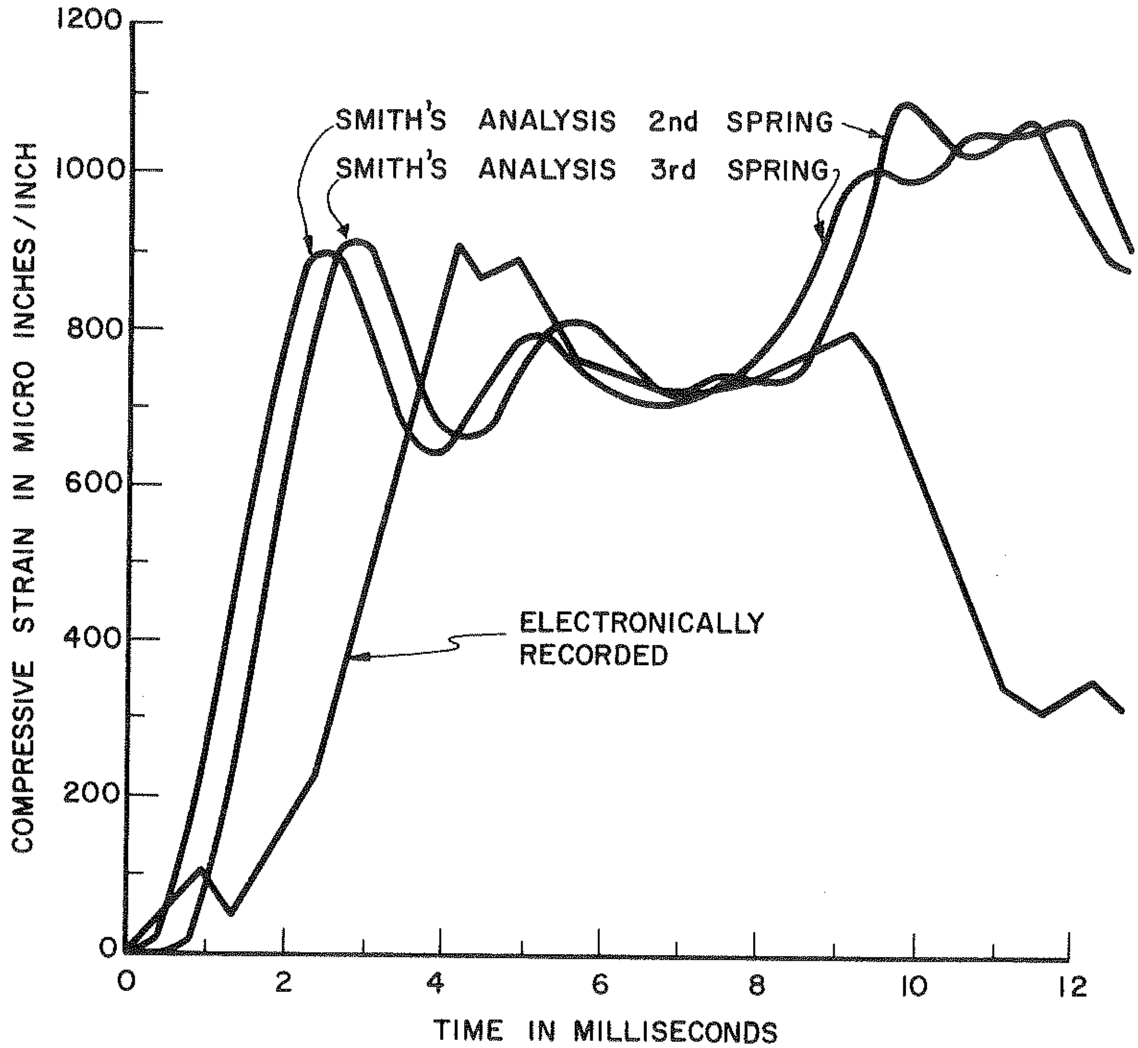


Figure 3.9

Velocity Versus Time



R_u = 200,000# R_u/Q = 2,000,000

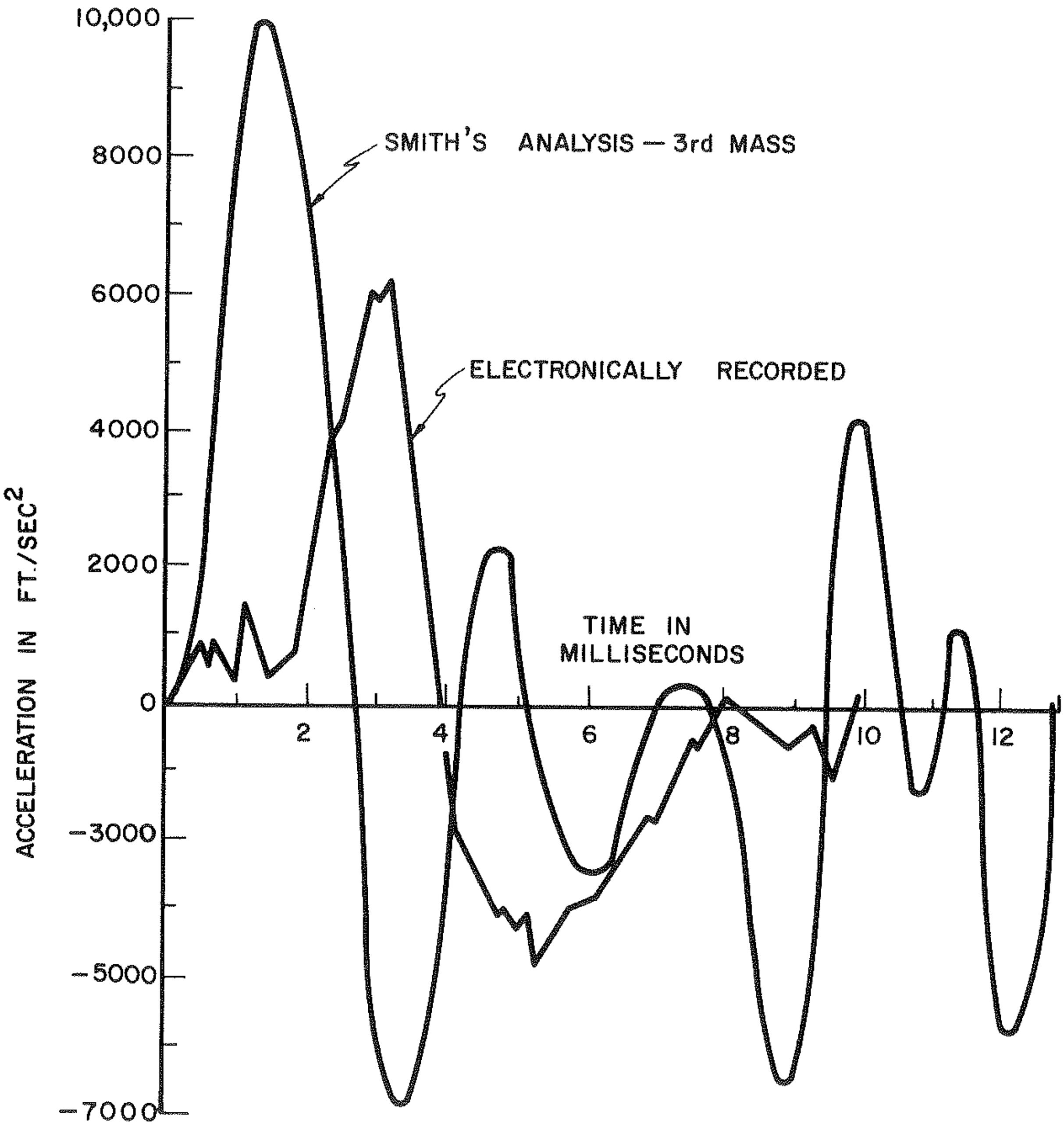
Damping and Quake Values: J = 0.15, J' = 0.05, Q = 0.1

Resistance Distribution: Point = 69% R_u, Side = 31% R_u

Set = 0.598 in/blow

Figure 3.10

Strain Versus Time



R_u = 2000,000# R_u/Q = 2,000,000

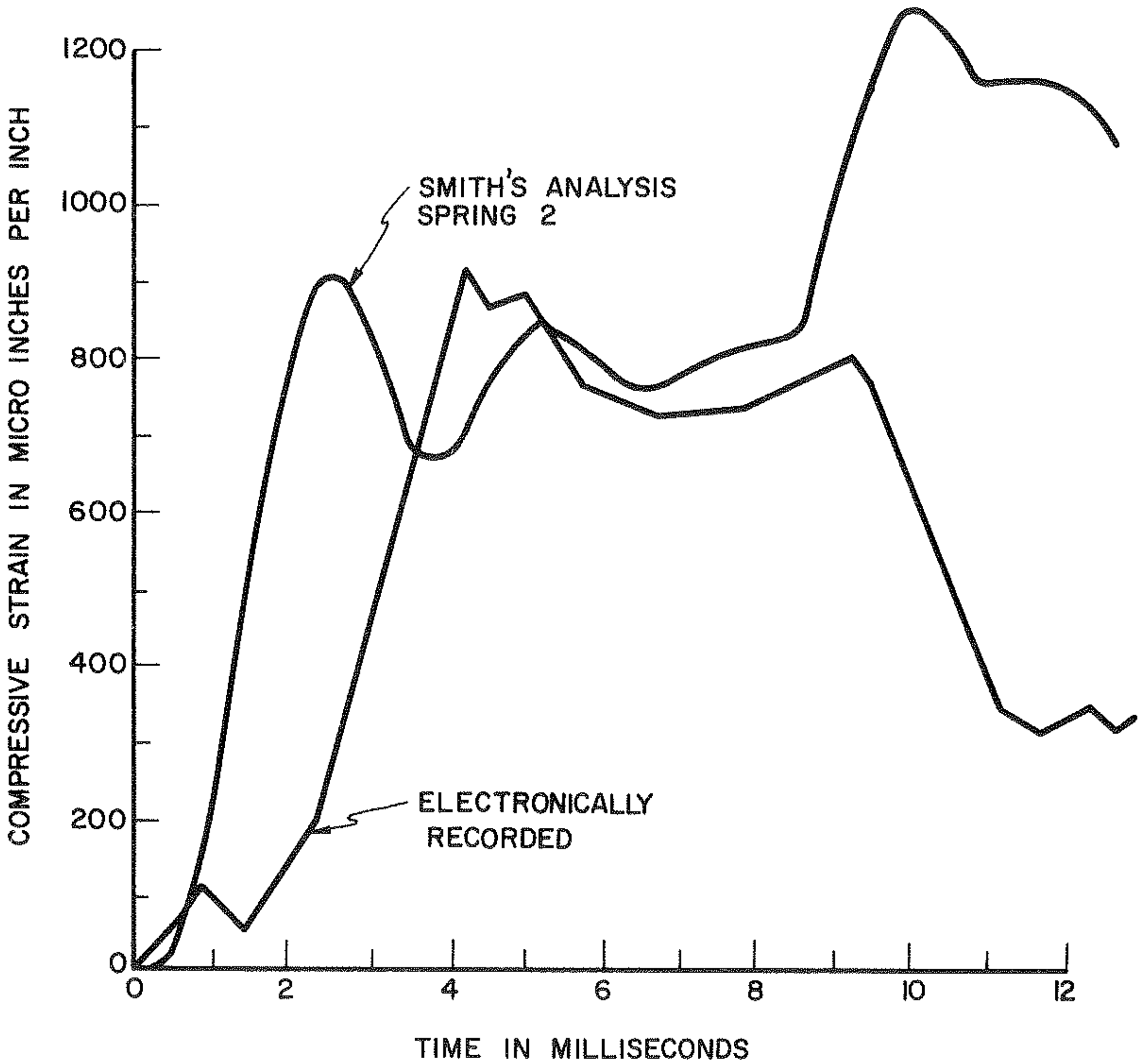
Resistance Distribution Point = 69% R_u Side = 31% R_u

Damping and Quake Values J = 0.15 J' = 0.05 Q = 0.1

Set = 0.598 in/Blow

Figure 3.11

Acceleration Versus Time



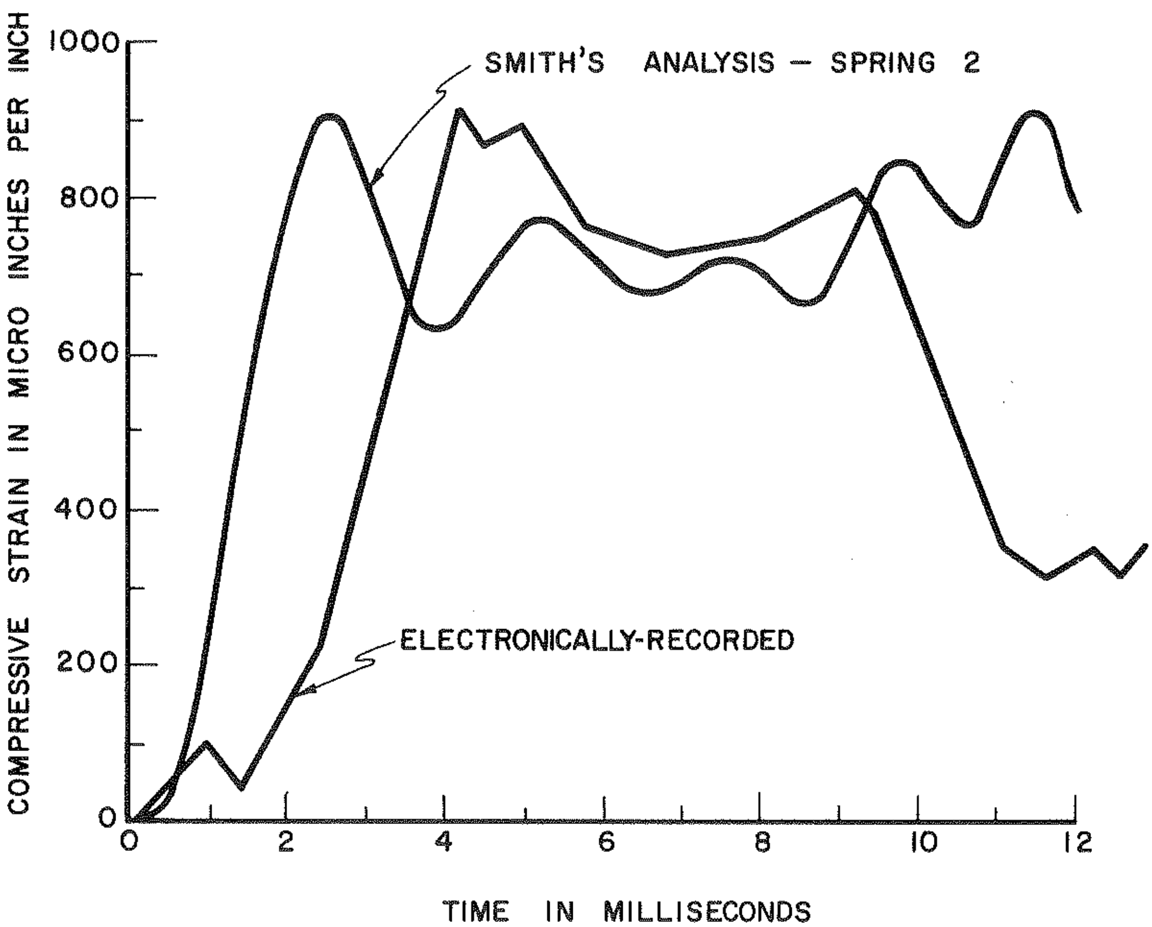
R_u = 200,000#　　　R_u/Q = 2,000,000

Resistance Distribution　Point = 69% R_u
　　　　　　　　　　　　Side = 31% R_u

Damping and Quake Values　J = 0.30
　　　　　　　　　　　　　J' = 0.10
　　　　　　　　　　　　　Q = 0.10

Figure 3.12

Strain Versus Time



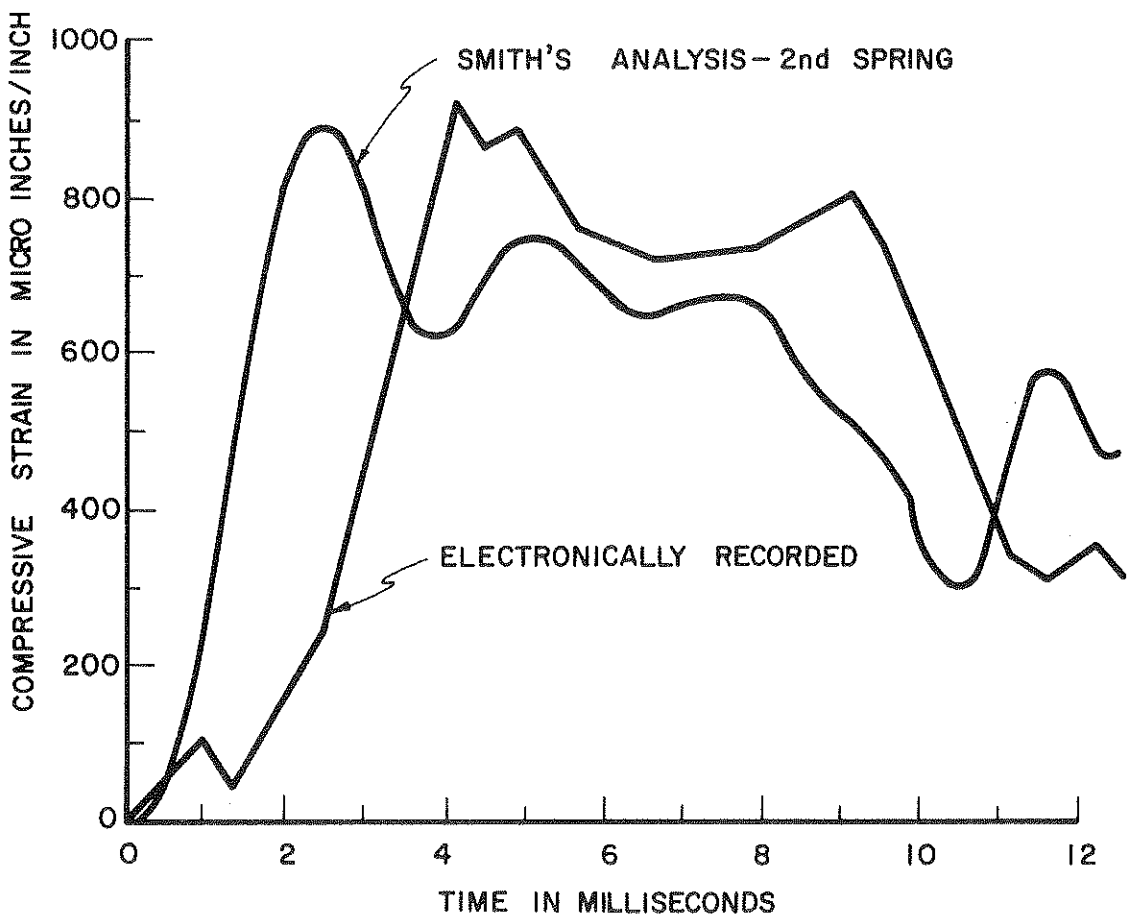
R_u = 200,000# R_u/Q = 2,000,000

Resistance Distribution   Point = 69% R_u
Side = 31% R_u

Damping and Quake Values   J = 0.075
J'= 0.025
Q = 0.1

Figure 3.13

Strain Versus Time



R_u = 200,000# R_u/Q = 2,000,000

Damping and Quake Values: J = 0.015, J' = 0.005, Q = 0.1

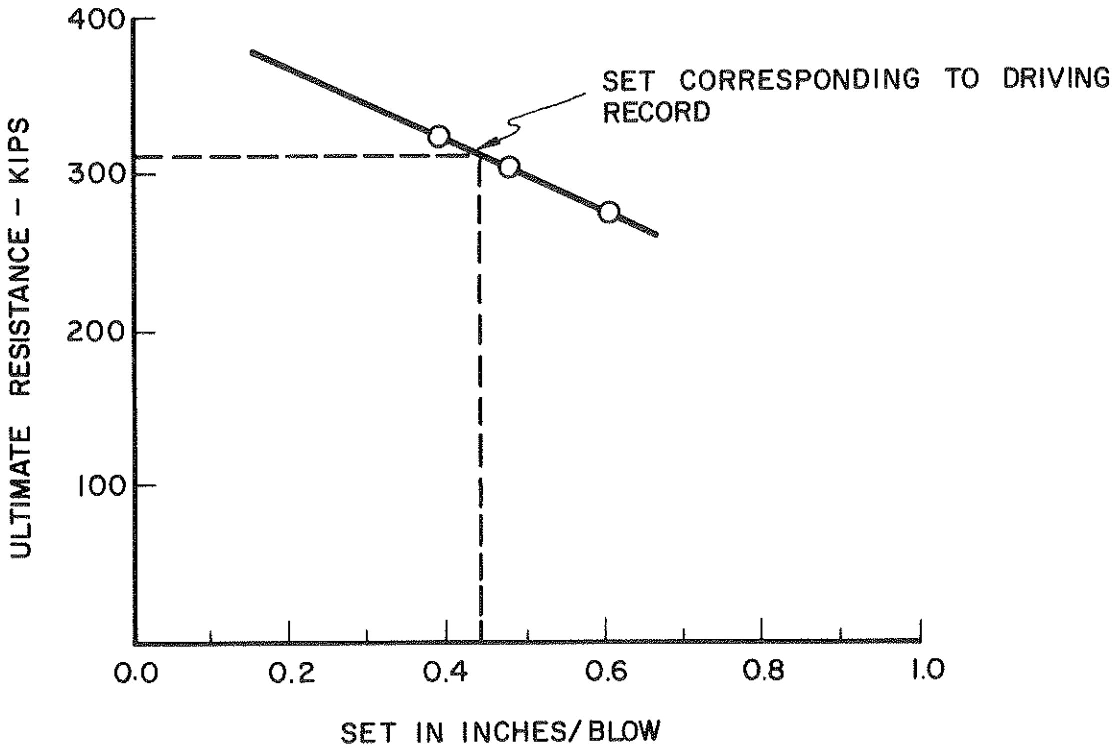
Resistance Distribution: Point = 69% R_u, Side = 31% R_u

Not Carried to Set by Computer

Set > 0.83 Probably > 1.0

Figure 3.14

Strain Versus Time



Resistance Distribution: Point = 69% R_u, Side = 31% R_u

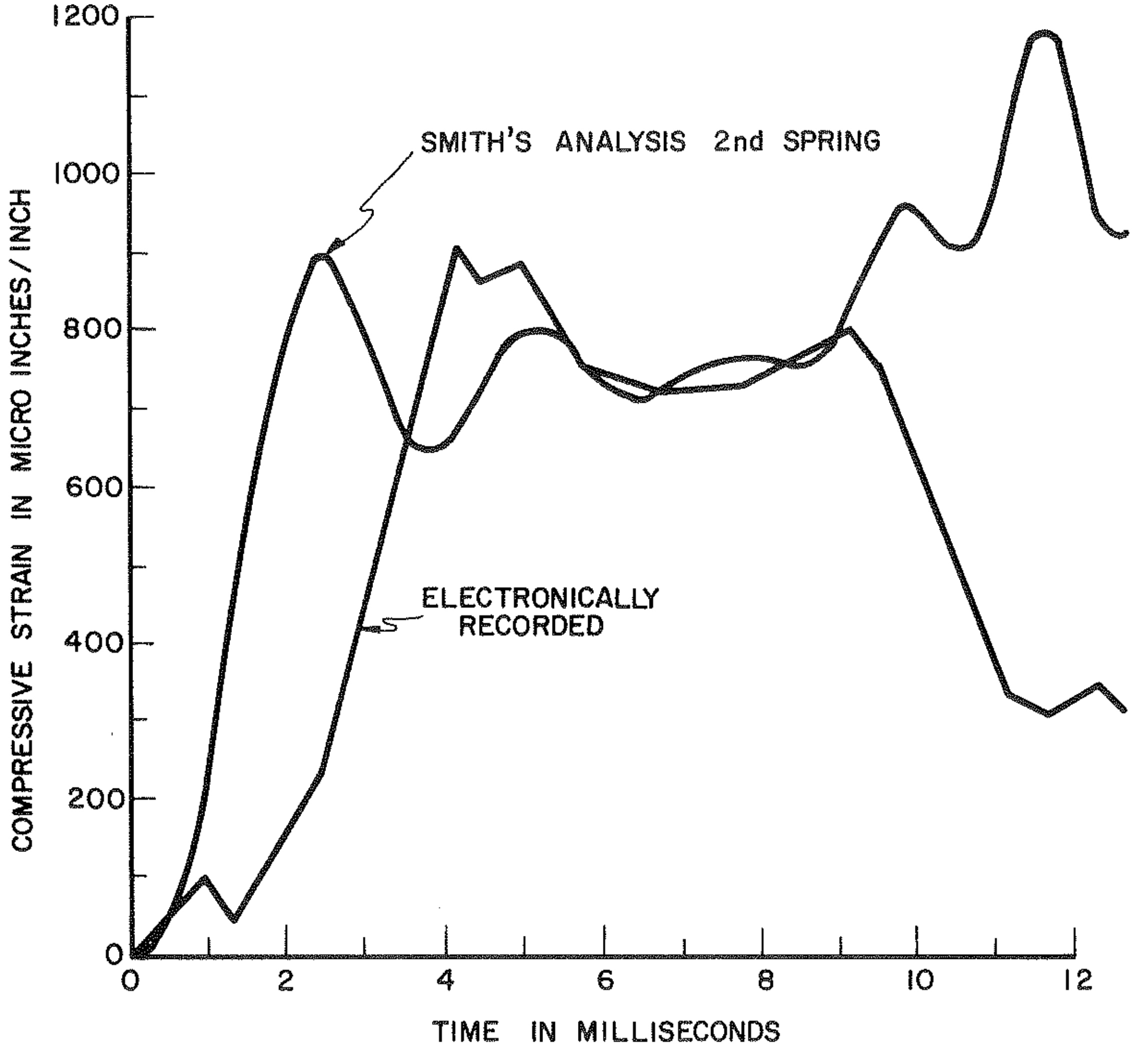
Damping and Quake Values: J = 0.015, J' = 0.005, Q = 0.1

R_u = 312 Kips

Set = 0.44 inch/blow

Figure 3.15

Set Versus Ultimate Resistance



R_u/Q = 3,050,000 R_u = 305,000

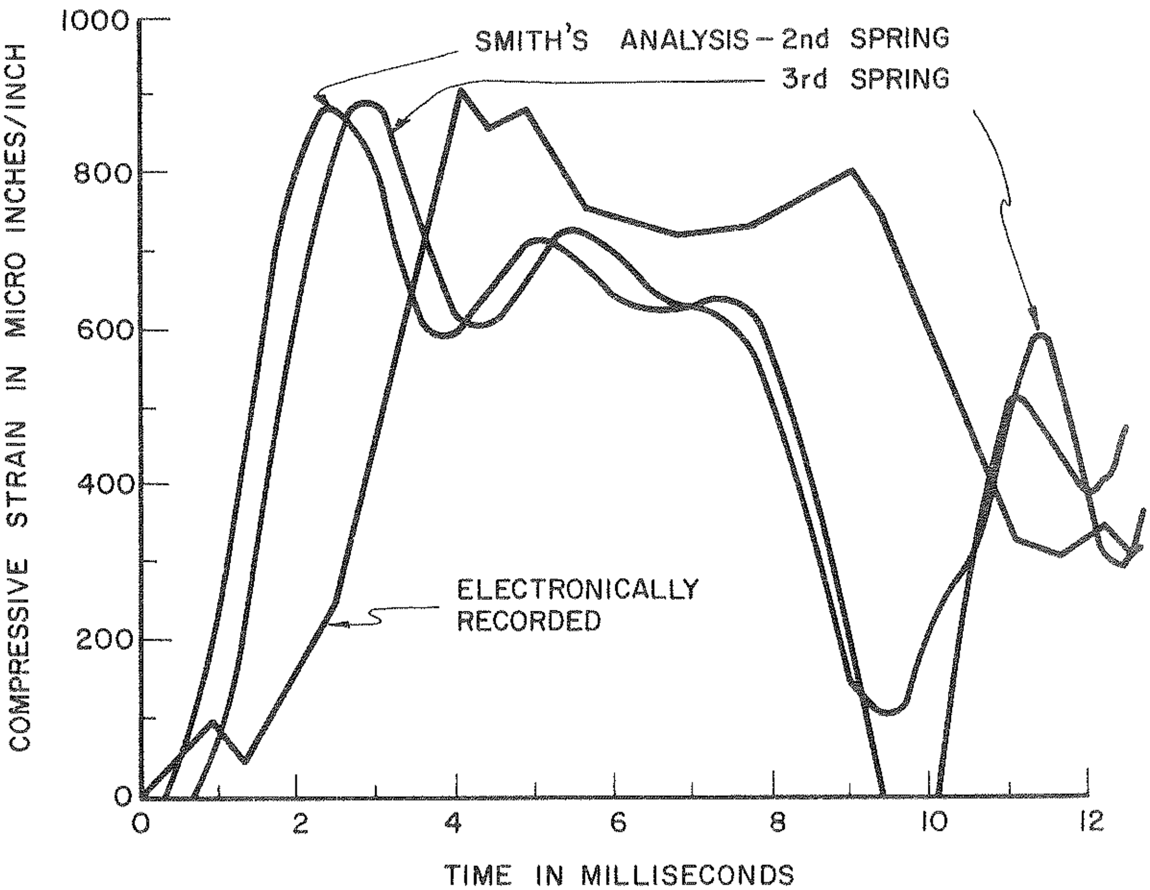
Damping and Quake Values J = 0.015, J' = 0.005, Q = 0.1

Resistance Distribution: Point = 69% R_u, Side = 31% R_u

Set = 0.48 in/blow

Figure 3.16

Strain Versus Time



R_u = 200,000# R_u/Q = 500,000

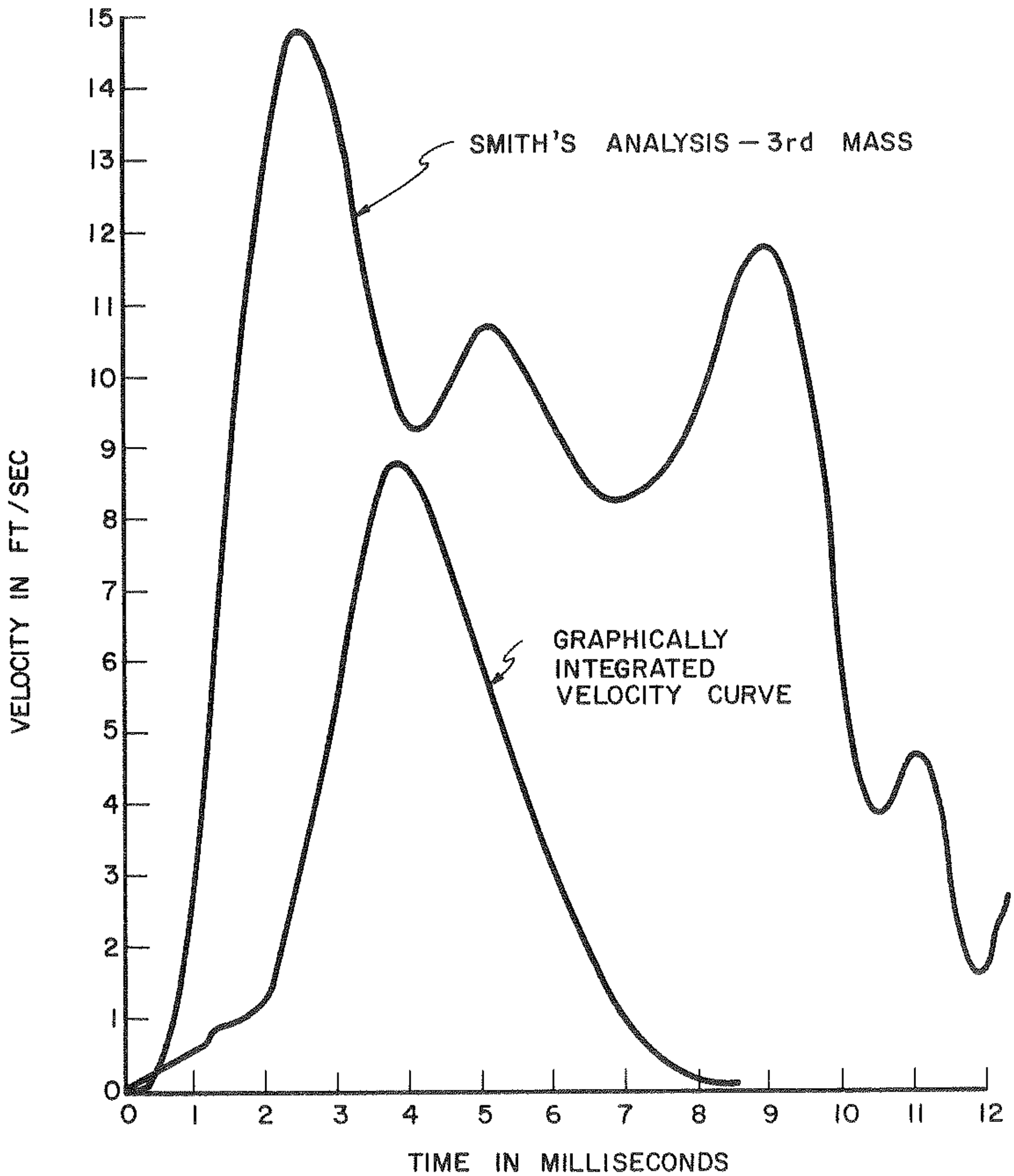
Resistance Distribution Point = 69% R_u
Side = 31% R_u

Damping and Quake Values J = 0.015
J' = 0.005
Q = 0.4

Set = 0.58 in/blow

Figure 3.17

Strain Versus Time



$Q = 0.4$ $R_u/Q = 500{,}000$ $R_u = 200{,}000$

DAMPING $\begin{cases} J = 0.015 \\ J' = 0.005 \end{cases}$

RESISTANCE DISTRIBUTION $\begin{cases} \text{POINT} = 69\% \, R_u \\ \text{SIDE} = 31\% \, R_\mu \end{cases}$

Figure 3.18

Velocity Versus Time

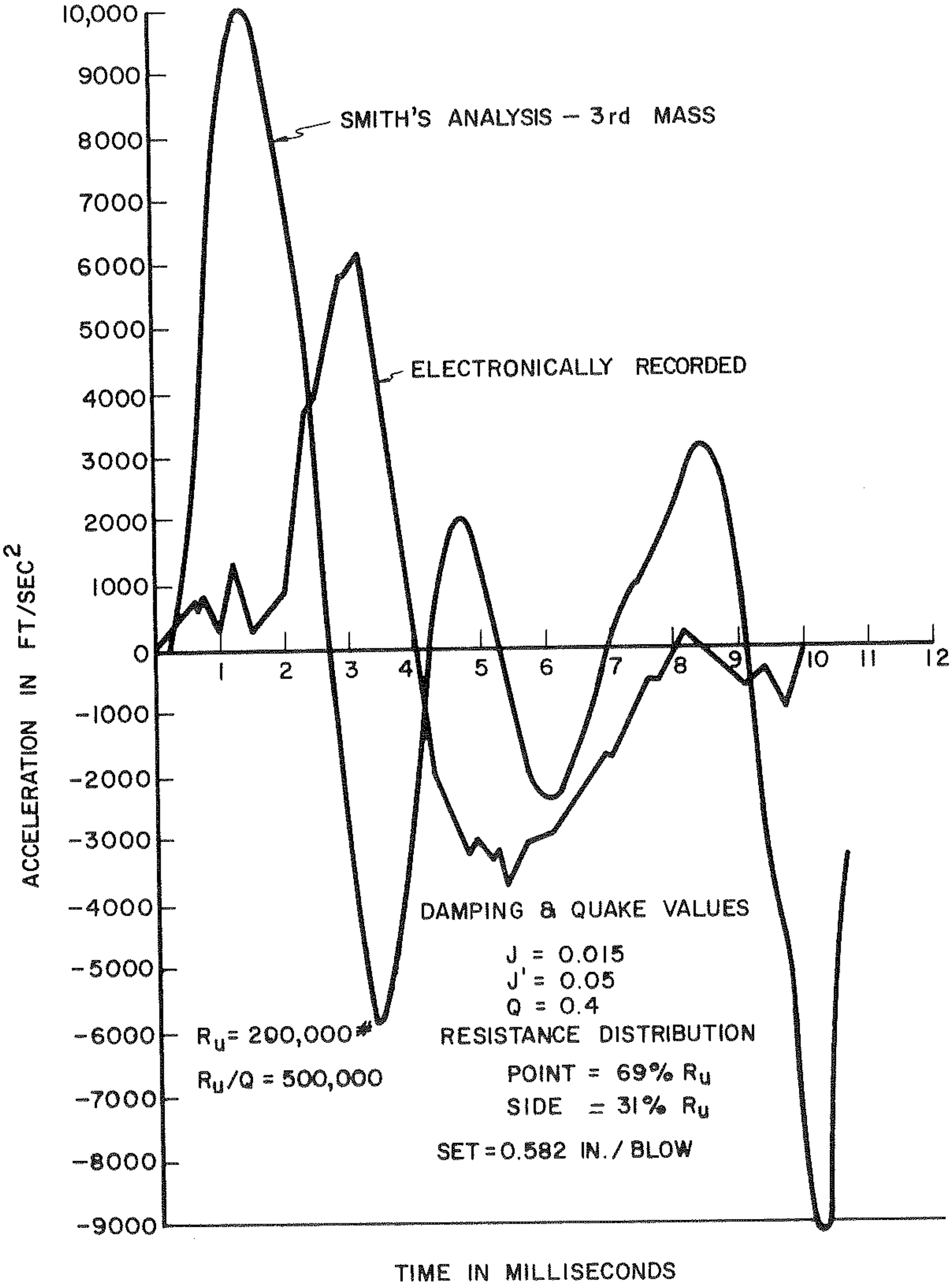


Figure 3.19

Acceleration Versus Time

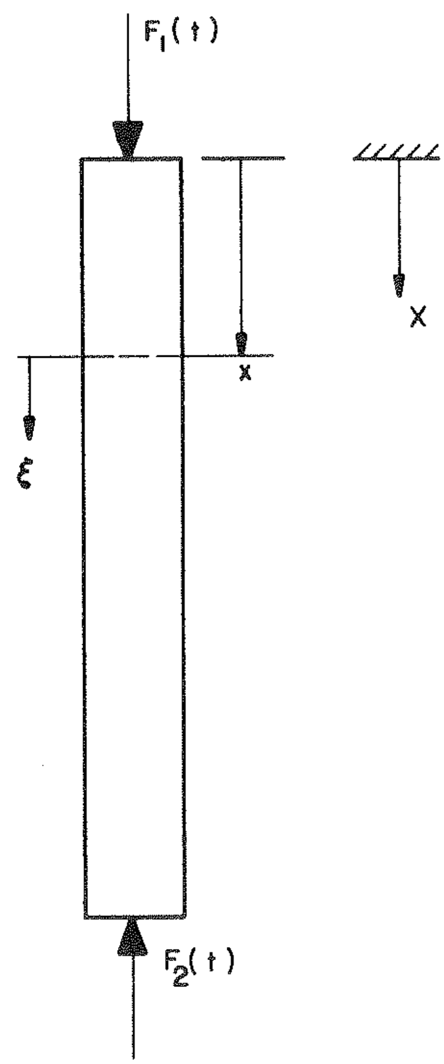


FIGURE 4.1

Idealized Pile Model

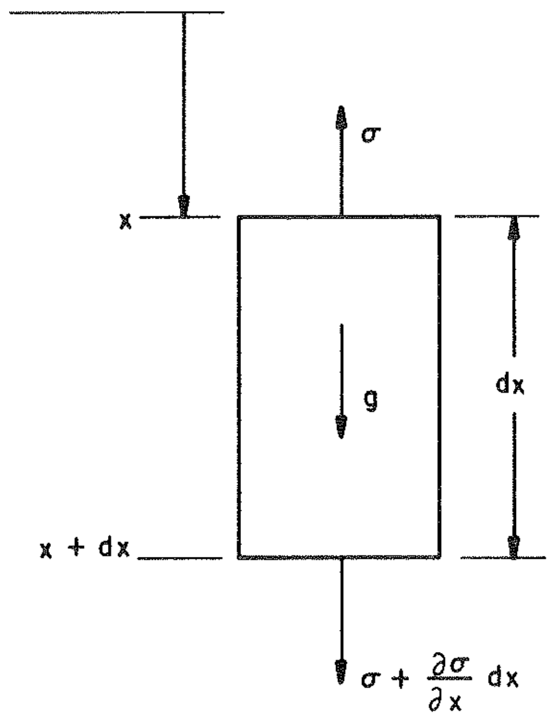


FIGURE 4.2

Stresses Acting on Differential Element

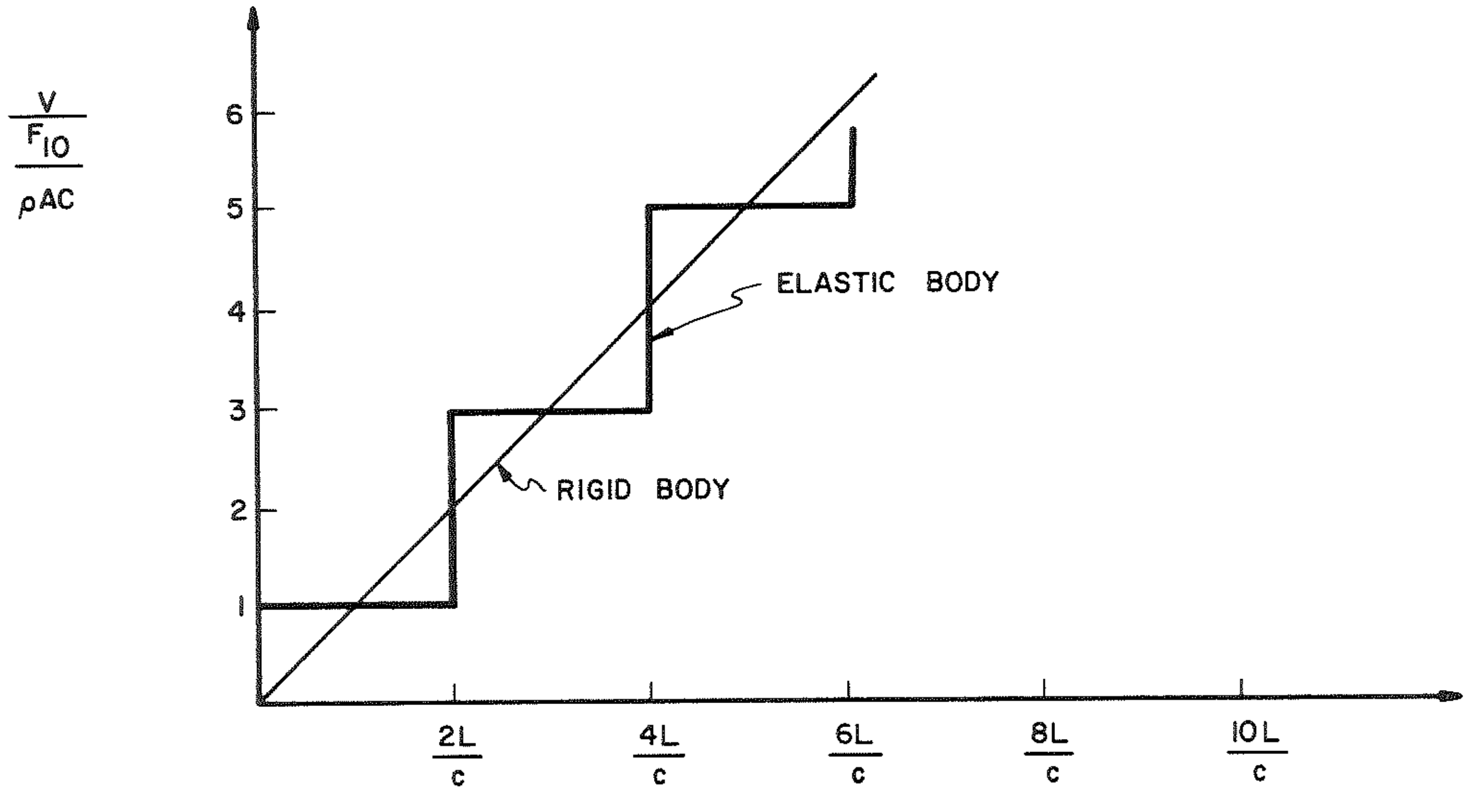


FIGURE 4.3

Velocity under a step input force
at the pile top (gravity effect omitted)

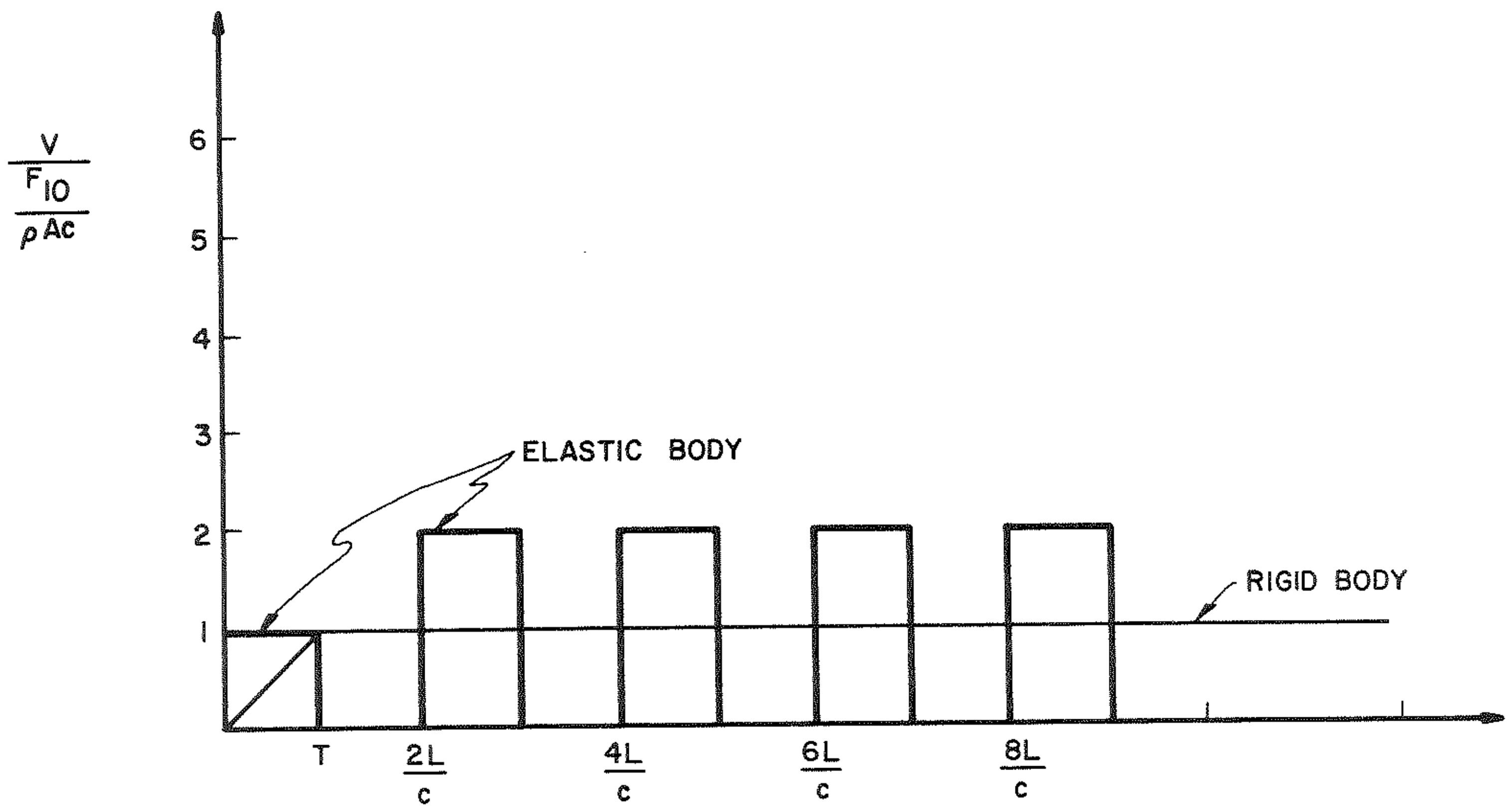


FIGURE 4.4

Velocity under a top force pulso of duration T (gravity effect omitted)

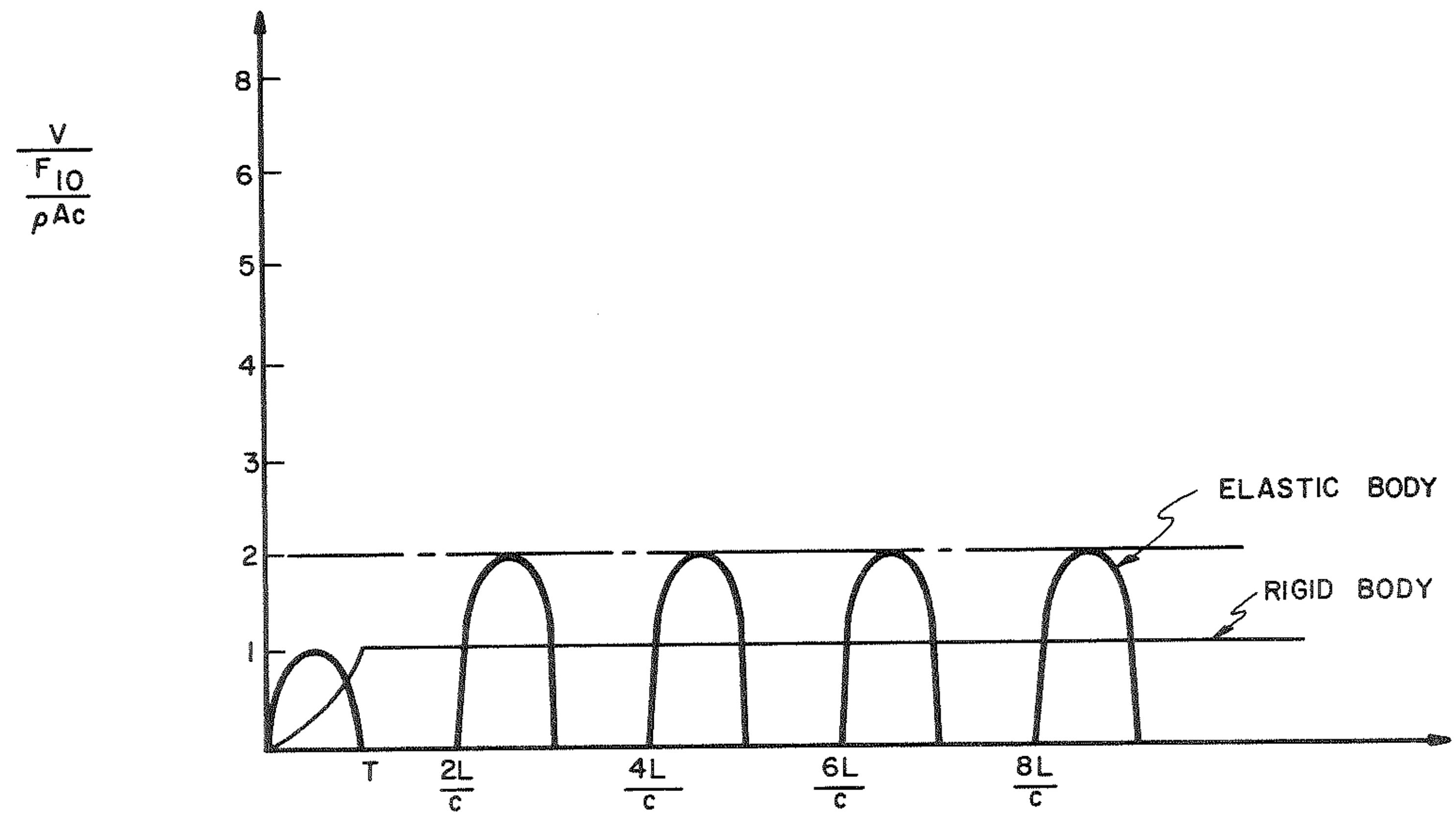


FIGURE 4.5

Velocity Response of Pile top to a ton force pulse of half-sine-wave form, duration T (Gravity effect neglected)

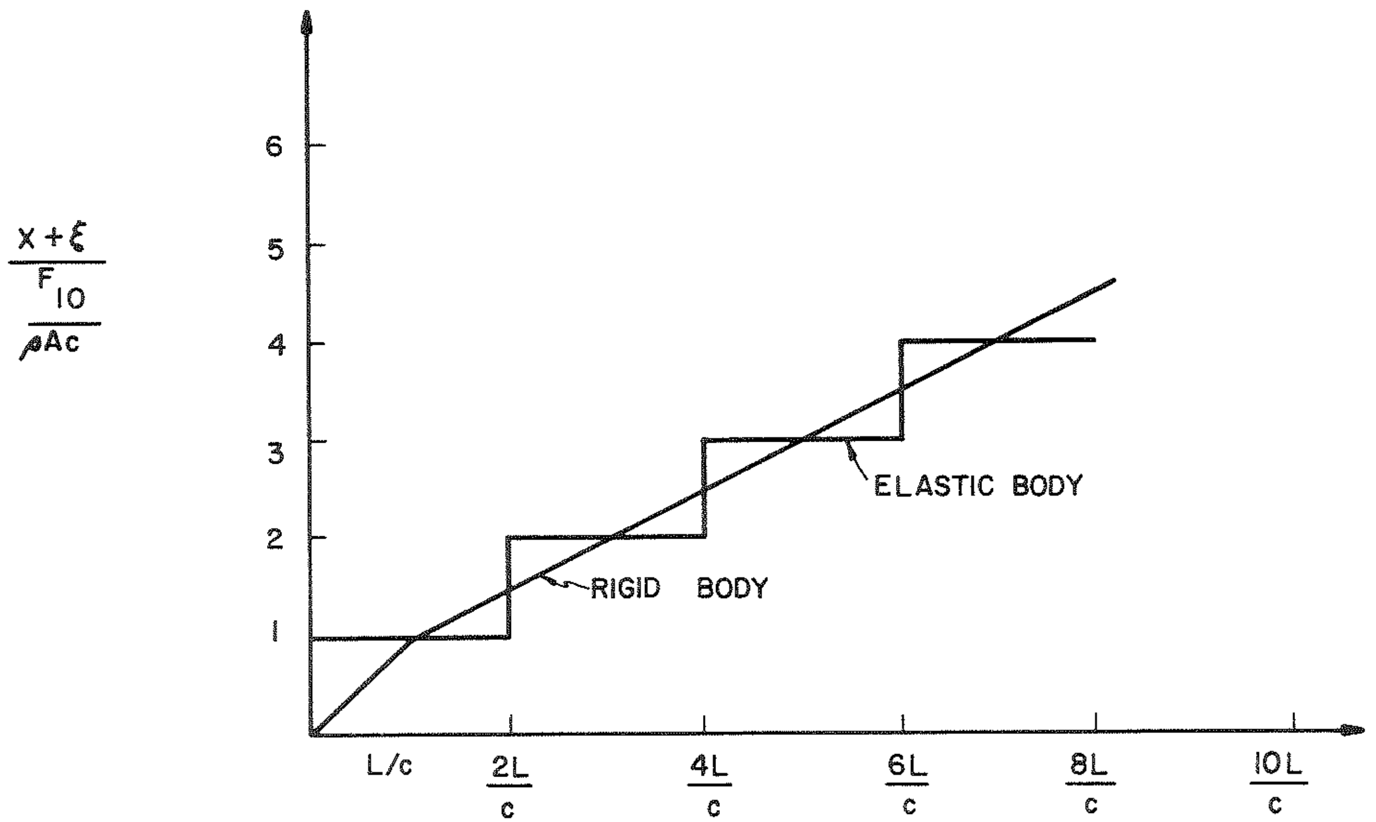


FIGURE 4.6

Displacement at top of pile under Dirac Impulse blows at top and bottom of pile

$(F_{20} = \frac{1}{2} F_{10})$

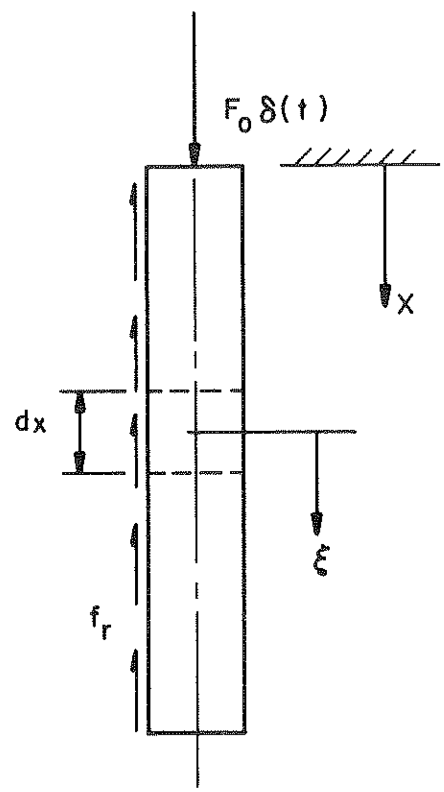


FIGURE 4.7

Idealized Pile Model

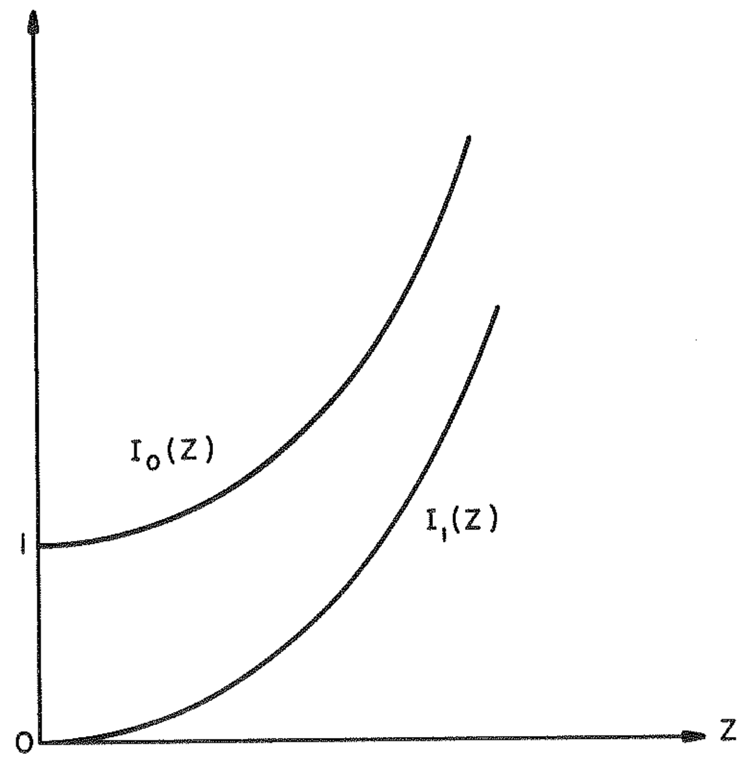


FIGURE 4.8

Hyperbolic Bessel Functions

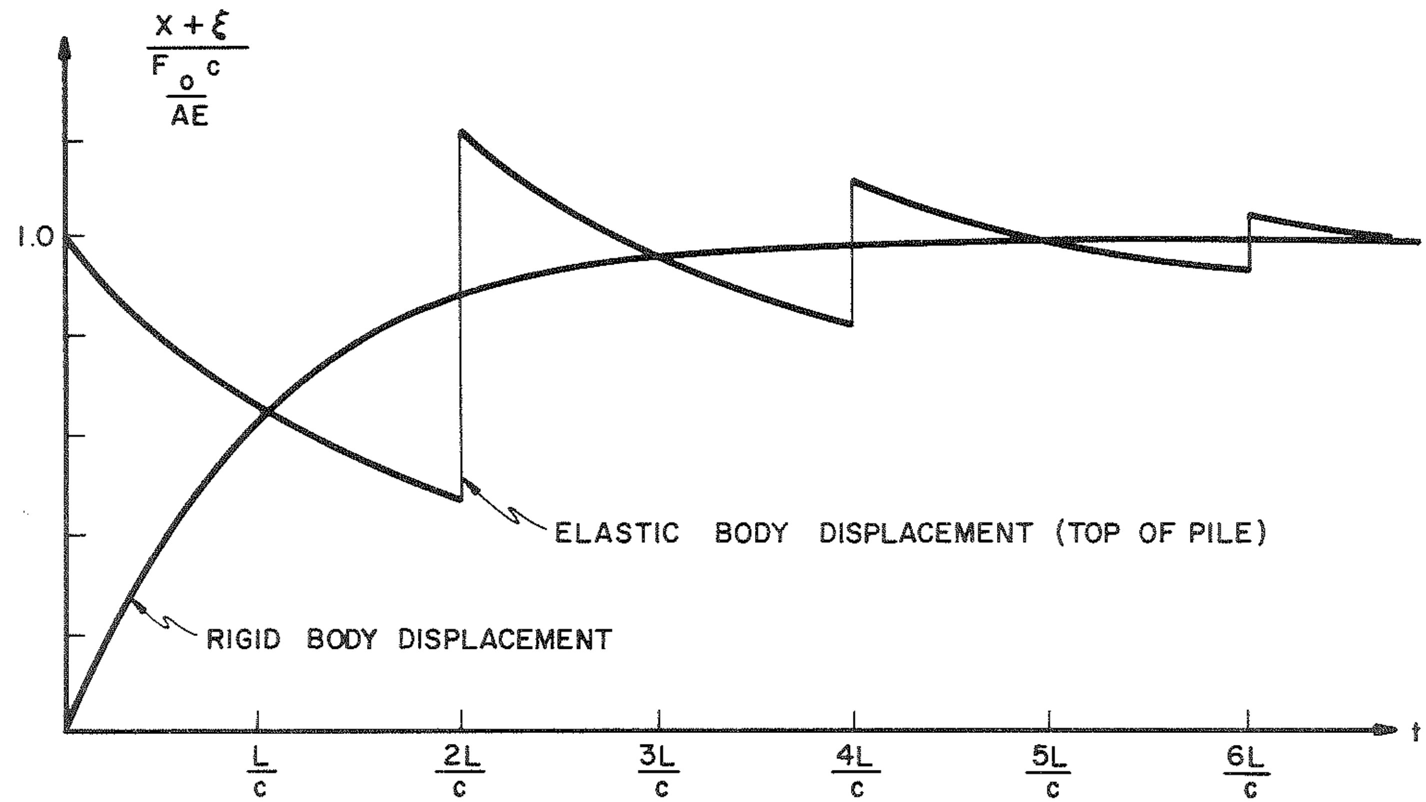


FIGURE 4.9

Pile Displacement Responses
to Instantaneous Pulse

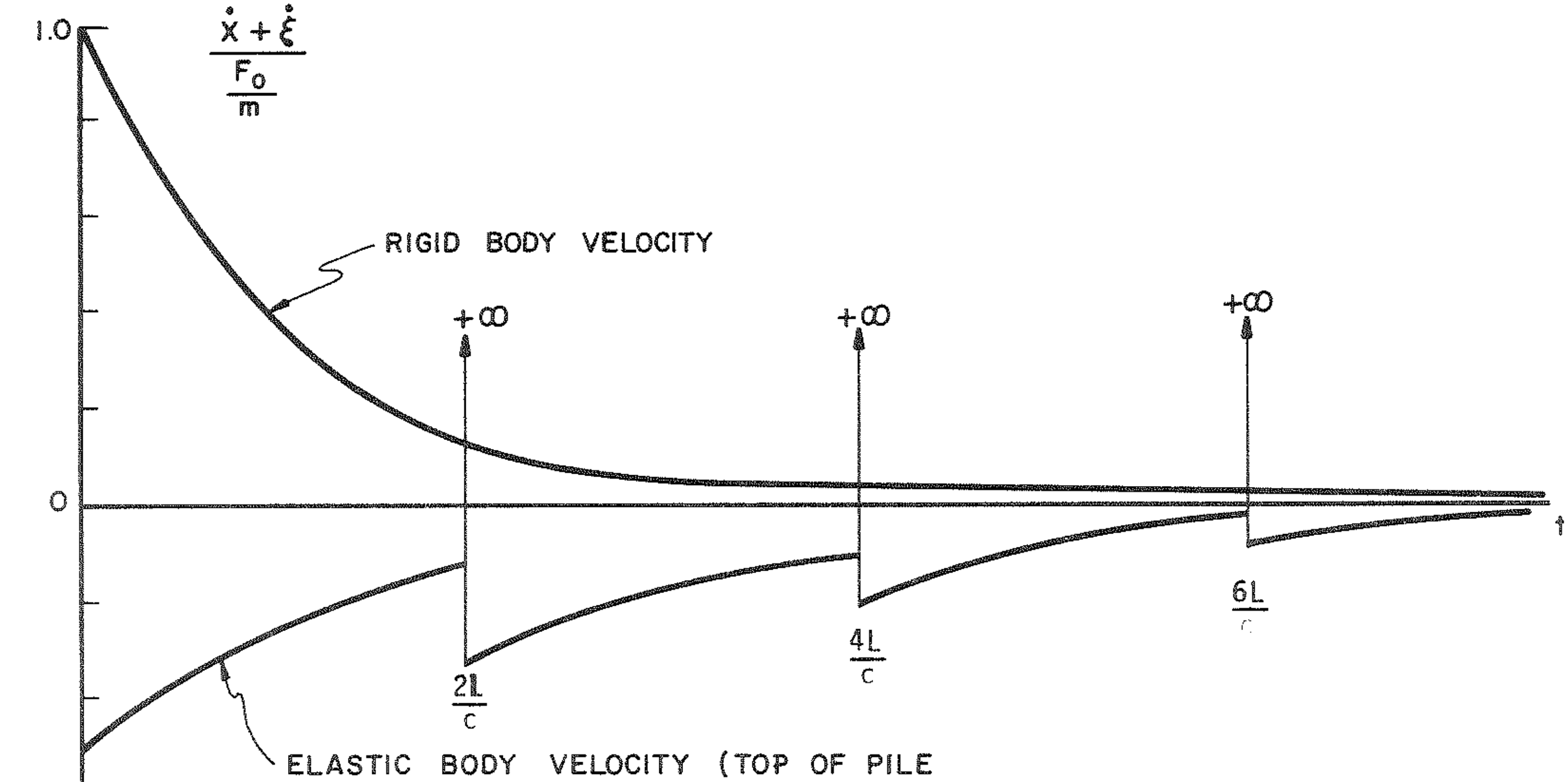


FIGURE 4.10

Pile Velocity Responses
to Instantaneous Pulse

A P P E N D I X

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