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Determination of wave equation soil constants from the standard penetration test

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ABSTRACT: The Standard Penetration Test is the single most widely used subsurface investigation method. Since the test is basically dynamic it would be very useful if it could be used to obtain more rational measures of soil performance that simply blow count. Since the test is performed in an open drill hole there is no shaft resistance along the sides of the drill rod. In this case it is possible to calculate the force and acceleration at the top of the rod. Measurements have been made in a variety of soil types and the sampler response calculated. Smith model Wave Equation constants were determined from the toe resistance, velocity and displacement by using unconstrained minimization routine to minimize the difference between the toe resistance calculated from top measurements and the same variable obtained from the Smith soil representation. Results obtained from the proposed method are also presented.

INTRODUCTION

The Standard Penetration Test is the most widely used subsurface investigation tool in the United States. In many other parts of the world it is also dominant. It has the advantage of producing a soil sample and it has been used for many years generating a large volume of experience. While the fact that the test is dynamic has been seen as a disadvantage by engineers, this is considered to be advantageous in some applications such as the analysis of Pile driveability or the evaluation of liquefaction potential. Since the test developed over a period of time prior to any standardization efforts it is still performed with a variety of equipment, techniques and procedures. This variability has been discussed extensively for more than two decades.

Energy transmission in the Standard Penetration Test has been studied with the principal goal of the work being to try to reduce the variability of the test. At least it has been found desirable to control or perhaps measure the energy transmitted to the top of the drill rod. With a knowledge of input energy it has been shown that the blow count could be adjusted to any desired standard energy (Schmertmann 1979). However, the measurement of energy proved to be difficult since it requires the measurement of both force and motion at the top of the drill string for its general determination. From practical considerations the only realistic motion measurement is acceleration and experience has shown that correct acceleration measurements are difficult to obtain in the case of steel-to-steel impact. Force measurements are also sometimes difficult. Hauge (1979) studied force and acceleration measurements on the SPT and showed that the acceleration at the top of the drill string can contain frequencies as high as 40-50 khz, within the range of the natural frequency of typical piezoelectric accelerometers. However, some modern accelerometers produce good quality measurements.

It is possible to take a completely different approach to evaluation of the Standard Penetration Test, avoiding energy calculation completely. If measurements of force and motion are made at the top of the drill string then force and motion can be calculated at the sampler as a function of time (Teferra 1977). It is hypothesized that such information could be of considerable value in the evaluation of soil properties. For instance, such measurements have an obvious application in the estimation of the shaft resistance of piles and also in the determination of dynamic resistance to driving. Such rather direct applications should require little calibration. Another interesting possibility is to use the force-time record determined at the sampler to make more reliable estimates of liquefaction potential. A method will be presented for determining wave equation soil constants from the calculated toe force and motion.

ANALYTICAL BACKGROUND

Consider the forces acting on the SPT rod during its operation. The drill rod extends down the drill hole with no resistance forces acting on the shaft during an impact by the ram. The penetration of the sampler is impeded by the resistance of the soil at the bottom of the drill hole. Thus, the only forces acting on the drill string are the impact forces and the sampler resistance forces. At the top of the rod the force and velocity are measured (actually obtained from measured acceleration). It will be shown that the force and motion at the bottom of the rod can be determined from the measured quantities at the top.

During an impact event the force in the rod at the measurement point can be divided into a downward and an upward traveling wave. The downward traveling force wave has been shown to be (Fischer 1984)

$$F\downarrow(t)=\frac{1}{2}[F_m(t)+\frac{EA}{c}v_m(t)] \qquad (1)$$

where $F_m(t)$ is the force measured at the top and $v_m(t)$ is the velocity measured at the same location as the force, both as a function of time, E is the modulus of elasticity of the rod material, A is the area of the rod and c is the velocity of wave propagation in the rod. The upward traveling force wave is

$$F\uparrow(t)=\frac{1}{2}[F_m(t)+\frac{EA}{c}v_m(t)] \qquad (2)$$

When the SPT is driven the impact induces a force at the top of the rod. This force propagates down the rod unchanged since there are no resistance forces along the shaft. When the force and associated motion arrive at the toe end they reflect back up the rod. Due to the motion, resistance forces are generated at the sampler and these forces also generate waves that travel up the rod. When they arrive at the top they effect the measurements.

Now consider these effects in a more quantitative manner. The upward traveling wave at the measurement point is generated by the effect of the downward traveling wave at the measurement point at a time 2L/c earlier and the toe resistance force L/c earlier. The rod can be considered as an impacted rod with a free bottom end with the resistance force superimposed on the other forces acting on the rod. When the downward traveling wave reaches the toe end it is reflected back up the rod with the opposite sign. Thus, the upward traveling force at the measurement point is equal to the negative of the downward traveling force at a time 2L/c earlier plus the toe resistance force at time L/c earlier. This can be expressed

$$F\uparrow(t+2L/c)=R(t+L/c)-F\downarrow(t) \qquad (3)$$

If the expressions of Equations (1) and (2) are substituted into Equation (3) and the resulting expression solved for the toe resistance force

$$R(t+L/c)=\frac{1}{2}[F_m(t)+F_m(t+2L/c)]+ \quad (4)$$

$$\frac{1}{2}\frac{EA}{c}[v_m(t)-v_m(t+2L/c)]$$

This relationship was derived based on the assumption of a uniform rod cross section with no shaft resistances. It can be expanded to include shaft resistance forces in addition to the tip forces included here (Rausche et al, 1985). In fact, the expression does not change but it is no longer possible to determine the source of the resistance forces from this closed form solution but only their total magnitude. If the rod is of variable cross section the same basic approach could be used, but from a practical point of view, it could best be calculated from wave propagation considerations using a discrete representation of the rod.

It is also useful to determine the velocity of motion at the toe. With velocity available both displacement and acceleration can be calculated. At the end of a free rod the particle velocity reflects with the same sign. During the reflection process the velocity at the end doubles. For a rod with resistance at the toe this velocity is reduced by the effect of the resistance force (proportional to the velocity effect). Therefore, in the case of the SPT, the velocity at the toe is

$$v(t+L/c)=2v{\downarrow}(t)-\frac{c}{EA}R(t+L/c) \quad (5)$$

where $v{\downarrow}$ is the downward traveling velocity wave. If the expression for the downward traveling velocity wave, at the measurement point, is substituted into Equation (5)

$$v(t+L/c)=[\frac{c}{EA}F_m(t)+v_m(t)]-\frac{c}{EA}R(t+L/c) \quad (6)$$

This expression can be integrated or differentiated to obtain the toe displacement or acceleration, respectively.

DETERMINATION OF WAVE EQUATION SOIL CONSTANTS

A systematic approach to determining soil parameters for use in wave equation analysis will be proposed. An automatic procedure will be presented to obtain the constants for the usual Smith soil model. Other soil models could be used just as easily. The Smith model is shown in Figure 1. The toe resistance force can be represented analytically as the sum of three separate resistance forces

$$R_{cT}=R_a+R_d+R_s \quad (7)$$

where R_a is the inertia or acceleration dependent resistance, R_d is the dynamic or velocity dependent resistance, and R_s is the static or displacement dependent resistance. The subscript cT denotes that the resistance force is calculated from the acceleration, velocity, and displacement determined at the toe from the top measurements. The acceleration dependent portion can be written

$$R_a(t)=ma(t) \quad (8)$$

where m is the mass of the material in the sampler. The dynamic portion is given by

$$R_d(t)=Jv^{N}(t) \quad (9)$$

where J is the viscous damping constant and N

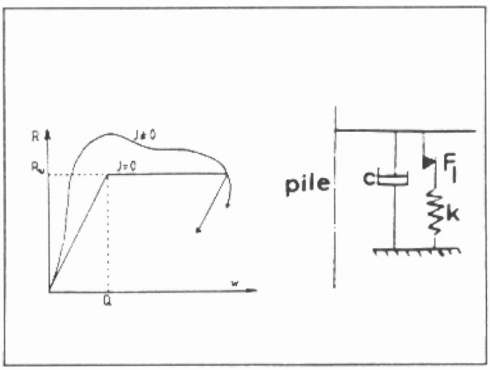

Figure 1: Smith Model

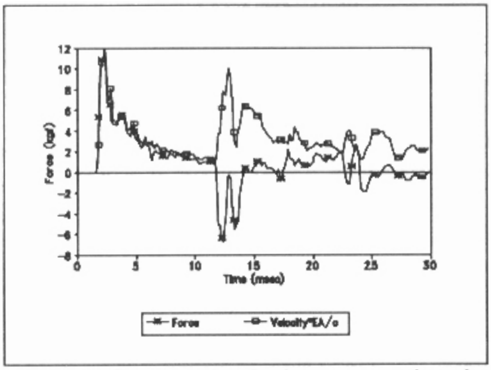

Figure 2: Force and velocity measured at the top of the rod.

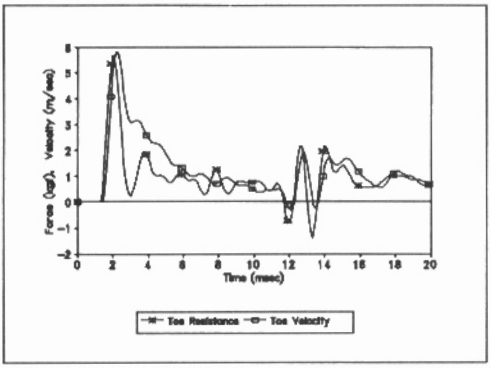

Figure 3: Toe resistance and toe velocity

is a power factor. Both of these constants are assumed to be soil constants. Other velocity dependent representations could be used. The static portion of the resistance can be stated

$$R_s(t)=\begin{cases}\dfrac{R_u}{q}\delta(t), & if\ \ \delta<q\\ R_u, & if\ \ \delta\geq q\end{cases} \tag{10}$$

where R_u is the ultimate static capacity as defined in Figure 1, q is the quake, and δ is the displacement.

Thus, for given values of m, J, R_u, q and N together with the values of the acceleration, velocity, and displacement obtained from the top measurement, R_{cT} can be calculated. If the proper values of the constants have been selected then R_{cT} should match the record calculated from the top measurements. The problem that must be solved is

$$\begin{aligned}&Find(m,J,R_u,q,N)\\&S.T.(R-R_{cT})^2\ \Rightarrow\ Min\end{aligned} \tag{11}$$

Sequential search techniques are available from Mathematical Programming to solve such problems. In this case a method was selected that does not require the calculation of analytic gradients to the function being minimized.

The method was applied to the measurement shown in Figure 2. This record was obtained from measurements taken at a site in the Denver area using the Pilcon hammer. The length below gages was 77.5 feet and the soil consists of clayey sand. The toe resistance (R) and the toe velocity that are calculated from this record are shown in Figure 3. The best obtained match between R_{cT} and the resistance calculated from the top measurements is shown in Figure 4. The corresponding values of m, J, R_u, q and N are 0.09 kg, 0.43 kgf-sec/m, 0.47 kgf, 0.61 mm and 1.0, respectively.

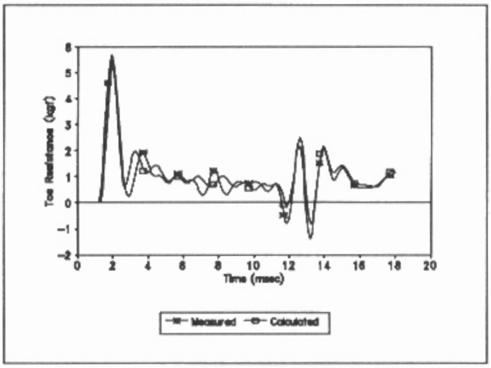

Figure 4: Comparison between RcT and the resistance calculated from top measurements.

CONCLUSIONS

A method to determine Smith model Wave Equation constants from dynamic measurements on the Standard Penetration Test has been presented. The method uses an unconstrained minimization routine to minimize the difference between the toe resistance calculated from top measurements and the one obtained from Smith soil model. An example using the method has also been presented showing reasonable results.

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