1981
BOULDER SEMINAR

Background of Capacity Interpretation
Using Dynamic Pile Measurements
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High Tension Stresses in Concrete Piles
During Hard Driving
By Garland E. Likins, Jr.

Load & Resistance Factor
Design of Piles
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By Garland E. Likins, Jr.
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CASE METHOD

by

Garland E. Likins and Frank Rausche

Introduction

The Case Method of pile capacity determination has been used with increasing frequency during the past ten years for both design and construction control of impact driven piles. As an increasing amount of static load test data became available for correlation the computation methods were modified, improved and refined.

The previous derivation of the fundamental Case Method equation has generally been preceeded by the assumption that the pile be treated as a rigid body. The expression resulting from the application of Newton's Second Law has then been modified without a complete derivation being presented with only a general reference to wave equation analysis. This type of presentation has caused the method to be criticized unjustly as being based on a rigid body assumption. It will be developed here in a brief way that will, perhaps, be more readable. Also the various modifications which are necessary to account for different soil and pile types are discussed.

In order to illustrate many of the characteristics of pile mechanics a typical force and velocity record for an impact driven pile is shown in Figure 1, where the velocity was obtained by integrating the measured acceleration over time. Also, for ease of plotting the velocity was multiplied by a constant, EA/c (Young's modulus times cross sectional area divided by the wave speed; all pile quantities). The time scale is given in milliseconds and in L/c units, i.e. in units of that time which a stress wave needs to travel along a pile of length L.

This record was obtained from a 100 ft. long concrete pile driven by a Kobe K-22 Diesel hammer. The static resistance as determined by a load test was 470 kips which is low compared to the force at impact. As a result the pile top force decreases at time 2L/c after impact, i.e.

when the impact wave returns after a (tension) reflection at the pile bottom.

2 Wave Mechanics

In the subsequent discussion it is assumed that the bar or pile is of uniform cross section. The derivation and solution of the one dimensional wave equation, a linear, second order differential equation, is available from other sources and will not be presented here.

Figure 1 shows one important phenomenon of wave mechanics, namely the fact that force and velocity at a point on a bar are proportional as long as stress waves at this point travel in only one direction. A wave in a rod which is free at top and bottom always has the same direction of velocity (while the sign of stress changes upon each reflection). The proportionality between the two curves is destroyed as soon as waves caused by soil resistance forces reach the pile top. However, for a completely free pile the top velocity due to a pile top force, $F_t(t)$, can be written

$$v_{t,F}\ (t) = \frac{c}{EA} F_t\ (t) \qquad 1$$

for $0 \leq t < \frac{2L}{c}$

At times later than 2L/c the effect of waves reflected from the pile bottom is felt at the top. Since a free top is assumed, the velocity will be doubled under reflection and will always be positive Therefore

$$v_{t,F}\ (t) = \frac{2c}{EA}\ \{\frac{1}{2} F_t\ (t) + F_t\ (t - \frac{2L}{c}) + F_t\ (t - \frac{4L}{c}) + \ldots\ldots\} \qquad 2$$

Resistance forces create a somewhat more complex wave behavior in a pile, since they, in general, act at intermediate locations along the pile. A suddenly applied force at such an intermediate location produces two waves. If the force, as in the case of soil resistance at a location x, $R_x(t)$, is directed upwards then the upwards traveling wave will be in compression and the downwards traveling one in tension. For reasons of continuity and equilibrium, together with the proportionality requirement between force and velocity, the forces in each wave must have a magnitude that is one half of the applied force. The velocities in both waves at the point of force application, x_i, are:

$$v_{t,Fi}(t) = \frac{1}{2}\frac{c}{EA} R_{x_i}(t) \qquad 3$$

Dealing again with a free pile on which this resistance force acts, the velocities will always be directed upwards. The pile top velocity when either of the two waves arrives and reflects will be twice the magnitude of that in Equation 3. The only difference between the two waves is in the arrival time. The upwards traveling wave arrives earlier than the other wave which is first reflected from the bottom.

Assuming that the distributed soil resistance is concentrated at n locations, x_i, $i = 1, \ldots, n$ (x_i measured from the top) whose magnitudes are R_i, $i = 1, \ldots, n$ and which are of the ideal plastic type such that

$$R_{x_i}(t) = R_i\, H(t - \frac{x_i}{c}) \qquad 4$$

where $H(t - a)$ is the Heavyside step function which is 0 for $t < a$ (negative arguments) and 1 for $t \geq a$. With $t = 0$ being the time of impact this soil resistance law implies that the resistance forces act only after the impact wave has reached their respective location and that they are constant thereafter.

The effect of the upwards traveling wave caused by $R_{x_i}(t)$ is felt at the top with a time delay x_i/c, and that of the downwards traveling wave with a delay $(2L - x_i)/c$ (after reflection at the bottom). This means that the first effect of $R_{x_i}(t)$ carried by the downwards traveling wave is felt together with the bottom reflected impact wave at time 2L/c after impact.

For all times one can write the top velocity due to the upwards traveling velocity caused by $R_{x_i}(t)$ as

$$v^u_{t,i}(t) = -\frac{c}{EA} R_i \{H(t - \frac{2x_i}{c}) + H(t - \frac{2x_i + 2L}{c}) + H(t - \frac{2x_i + 4L}{c}) + \ldots\} \qquad 5$$

remembering again that because of the reflection at the free top the wave velocity has doubled.

The downwards traveling wave causes a top velocity

$$v^d_{t,i}(t) = -\frac{c}{EA} R_i \{H(t - \frac{2L}{c}) + H(t - \frac{4L}{c}) + H(t - \frac{6L}{c}) + \ldots\} \qquad 6$$

The total velocity caused by the impact force, $F_t(t)$, and all n resistance forces R_i, i = 1, 2, ..., n can be written as

$$v_t(t) = \frac{c}{EA} \{F_t(t) + 2 \sum_{j=1}^{m} F_t(t - \frac{j2L}{c})$$

$$- \sum_{i=1}^{n} R_i [H(t - \frac{2x_i}{c}) + \sum_{j=1}^{m} H(t - \frac{2x_i + j2L}{c}) + \sum_{j=1}^{m} H(t - \frac{j2L}{c})]\} \qquad 7$$

where m indicates the time interval after impact:

$$\frac{m2L}{c} < t < \frac{(m + 1)\ 2L}{c}$$

3 Evaluation of Measurements

If this simplified soil model for the resistance force were correct then $v_t(t)$ would be equal to $v_M(t)$ (i.e. the measured velocity) when the measured force $F_M(t)$ was substituted for $F_t(t)$ in Equation. 7. Thus,

$$\frac{EA}{c} v_M(t) = F_M(t) + 2 \sum_{j=1}^{m} F_M(t - \frac{j2L}{c})$$

$$- \sum_{i=1}^{n} R_i \; [H(t - \frac{2x_i}{c}) + \sum_{j=1}^{m} H(t - \frac{2x_i + j2L}{c}) + \sum_{j=1}^{m} H(t - \frac{j2L}{c})] \qquad 8$$

The third term in Equation 8, the summation over all resistance forces is shown graphically in Figure 2. The first portion of the term is due to the first arrival at the pile top of the upward traveling wave due to the resistance at the location x_i. The full value is felt at $\frac{2x_i}{c}$ after impact and stays on for the remainder of the blow as shown by the solid bar in Figure 2. The second portion of the term is due to the same wave after reflections from the top and bottom. Thus, for the first and succeeding time intervals m the effects are felt 2L/c after the preceeding interval m-1, and are represented by the stripped bars in Figure 2. The third portion of the third term is due to the effect of the downward traveling wave. The wave arrives at 2L/c after impact and again remains on for the remainder of the blow. Due to subsequent reflections the same effect is felt every 2L/c. This portion is depicted by the white bars in Figure 2. Since all of these effects are of equal magnitude it can be seen from Figure 2 that the third term becomes

$$-\sum_{i=1}^{n} R_i [2\, m + H(t - \frac{2x_i + 2mL}{c})] \qquad 9$$

where m is the time interval. If the measured velocity is taken at any time t*

$$(m \frac{2L}{c} \leq t^* < (m+1) \frac{2L}{c})$$

and if the measured velocity at a time 2L/c later is subtracted, then the result is

$$\frac{EA}{c}\ \{v_M(t^*) - v_M(t^* + \frac{2L}{c})\} = F_M(t^*) + 2\sum_{j=1}^{m} F_M(t^* - \frac{j2L}{c})$$

$$-F(t^* + \frac{2L}{c}) - 2\sum_{j=1}^{m+1} F_M(t^* + \frac{2L}{c} - \frac{j2L}{c})$$

$$-\sum_{i=1}^{n} R_i[2\ m + H(t^* - \frac{2x_i + 2mL}{c}) - 2(m+1)$$

$$-H(t^* + \frac{2L}{c} - \frac{2x_i + 2(m+1)L}{c})] \qquad 10$$

Since the arguments of the Heavyside function terms both have the same value their effects cancel. In addition the first element of the fourth term becomes

$$-2\ F_M(t^* + \frac{2L}{c} - \frac{2L}{c})$$

and combines with $F_M(t^*)$ to produce $-F_M(t^*)$. The simplified expression then becomes

$$\frac{EA}{c}\ \{(v_M(t^*) - v_M(t^* + \frac{2L}{c})\} = -F_M(t^*) - F_M(t^* + \frac{2L}{c}) - \sum_{i=1}^{n} R_i\{2m - 2(m+1)\} \qquad 11$$

and with:

$$\sum_{i=1}^{n} R_i = R \qquad 12$$

i.e. the total resistance, yields

$$R = \frac{1}{2}\ \{(F_M(t^*) + F_M(t^* + \frac{2L}{c})) + \frac{EA}{c}\ (v_M(t^*) - v_M(t^* + \frac{2L}{c}))\} \qquad 13$$

For a uniform rod with

$$c = \sqrt{E/\rho} \qquad 14$$

and $M = LA\rho$ (ρ is the mass density, M the total pile mass) direct substitution gives

$$\frac{EA}{c} = \frac{Mc}{L} \qquad 15$$

Thus, Equation 13 can also be written as

$$R = \frac{1}{2}\{F_M(t^*) + F_M(t^* + \frac{2L}{c})\} + M\,\frac{v_M(t^*) + v_M(t^* + \frac{2L}{c})}{2L/c} \qquad 13a$$

which shows that the prediction R can be considered as an average of two force values 2L/c apart plus an inertia term using an average acceleration over the same time period. Equation 13a reduces to Newton's Second Law if L becomes small enough and the pile can be approximated by a rigid body. For larger lengths, L, where the pile must be considered an elastic rod, Equation 13a is valid.

While the pile elastic properties and the distribution of resistance forces were properly considered in the above derivation, the resistance force versus time variations were neglected. These variations exist because both dynamic resistance forces (soil damping) and unloading (pile rebound) occur.

Originally it was proposed to choose the time t*, yet to be decided upon, at the time when the pile top velocity became zero. Then, it was argued, soil damping forces would have become small. Results obtained in this way were usually conservative when compared with the ultimate capacity of the pile as determined in a static load test. There was an indication, however, that a correct correlation was to be made with a penetration related capacity (less than ultimate). This approach has been abandoned as it is difficult to use in construction control.

For steel pipe piles in granular soils, empirical correlation with ultimate capacity was best when choosing t^* at the time when the first relative maximum of velocity was reached (here called the time of impact).

4 Derivation of Damping Estimate

Equation 13a has been derived using a very simple model for soil behavior. An improvement in this model and a commonly used approximation is to assume that the forces R_i are really made up of two portions. The first portion is due to static resistance forces $R_{i,s}$ and their sum RS is then the actual failure load. The second portion $R_{i,d}$ is due to dynamic resistance forces (damping) which are usually treated as being proportional to velocity. The sum of these dynamic forces RD is important only during the driving of he pile and is of no further practical value. Thus, the total driving resistance RT can be broken up into two distinct portions

$$RT = RS + RD \qquad 16$$

In order to more closely approximate the pile's static capacity from dynamic measurements, an estimate of the total damping forces RD must be made.

Because the damping effects of soil and unloading due to pile rebound diminish the force and velocity waves in a rather short time period, only the first 2L/c period is usually available for estimation of the maximum damping resistance RD. Further, for most pile types the majority of both the static soil resistance and dynamic forces have their origin at the pile tip rather than in frictional side resistance. This hypothesis has been confirmed by using a wave equation analysis on the dynamically measured data and also through strain gages located along the pile length, where readings were taken during the static load tests. The bottom velocity of the pile for the free pile solution after the impact arrives and reflects is

$$v_B(t) = 2\, v_t(t - \frac{L}{c}) \qquad 17$$

for $\frac{L}{c} \leq t \leq \frac{3L}{c}$.

The effect of the downwards traveling wave caused by R_{x_i} (t) on the bottom velocity is given by twice the magnitude of Equation 3 due to reflection.

$$v^d_{B,i}\ (t) = -\frac{c}{EA}\ R_i\ H(t - \frac{L - x_i}{c}) = -\frac{c}{EA}\ R_i \qquad 18$$

for $\frac{L}{c} \leq t \leq \frac{3L}{c}$.

The pile top velocity characteristically shows a relative maximum in the beginning of the blow (impact) and then diminishes in magnitude with time. In cases of very easy driving, the large tip penetrations and tensile reflections cause a large increase in pile velocity at 2L/c after impact. In several cases the top velocity after the impact reflection from the weak tip can become significantly larger than the pile velocity at impact. Because piles are rarely considered acceptable for capacity in such easy driving, they are not of great importance. However, in the cases where they have been encountered by the project the velocity estimates are satisfactory. The bottom velocity will reach a relative maximum value at time $t = t_{max} + \frac{L}{c}$ (t_{max} is the time of impact) and is given by

$$v_{B,max}\ (t_{max} + \frac{L}{c}) = 2v(t_{max}) - \frac{c}{EA} \sum_{i=1}^{n} R_i \qquad 19$$

(The effect of the upward traveling wave caused by R_{x_i} will be zero at time L/c because all Heavyside step functions $H(t - \frac{L + 2x_i}{c})$ are zero).

If the damping force is treated as proportional to this bottom velocity then the maximum damping force becomes

$$RD = bv_B = J_c\ \frac{EA}{c}\ vB \qquad 20$$

where J_c is a dimensionless damping parameter (see Appendix A). Use of Equation 13a with t^* equal to t_{max} gives the maximum driving resistance RT.

Rearranging Equation 16 and using Equations 1, 12, 19 and 20 gives the maximum static resistance RS as

$$RS = RT - J_c(F_t(t_{max}) + \frac{EA}{c} v_t\ (t_{max}) - RT) \qquad 21$$

Use of Equation 21 can then be made with the measured force and velocity functions of time and the actual failure load of the pile as determined from a static load test (LT) to determine the correct value of J_c for any particular pile

$$J_c = (RT - LT)/(F_t(t_{max}) + \frac{EA}{c} v_t(t_{max}) - RT) \qquad 22$$

The Heavyside step functions used in the preceeding derivations serve only to simplify the algebraic manipulations. The same equations (13 and 21) for capacity and damping could be obtained using an elastic plastic resistance law (versus the rigid plastic one used here) as commonly found in the classical lumped mass, numerical wave equation analyses now used extensively in the United States. The only necessary restriction is the displacement at the time E_x for location x below the measurements

$$t_x = t^* + \frac{x}{c} \qquad 23$$

(where t^* is the first time in Equation 13a) is greater than the soil quake q_x (the displacement where the elastic plastic soil law becomes plastic) at location x. The time t^* can be increased if necessary to allow time for extra displacement and the condition to be satisfied.
In most cases the time t_{max} does indeed provide the maximum resistance RT. In some cases (using little cushioning or steel capblocks, small hammers used for high resistances or other very hard driving cases, or if the soil quakes are large, etc.) this maximum RT will occur at some time after the peak input velocity. Automatic searches for the maximum RT capacity are now included as standard procedure.

Later, it was found, when analyzing data from piles whose capacity was large compared to the hammer driving capability, or which had long lengths L (t^* + 2L/c was then at a very late time) that rebound sometimes occurred before 2L/c after impact implying that unloading had occurred. A technique to estimate the amount of unloading has since been included (see Appendix B).

The only assumptions used in the derivations above for ultimate pile capacity are

a) the pile is a uniform elastic rod with length much greater than diameter

b) that the soil quake is exceeded at every point along the pile

c) static resistance is related to pile displacement and damping resistance to pile velocity

Violation of condition b will result in very low observed permanent set per blow. If a meaningful rate of penetration is not achieved, the Case Method (or any other dynamic analysis technique, i.e. dynamic formula, wave equations, CAPWAP) can only be expected to indicate the soil resistance actually mobilized.

5 Results of Damping Approach

For most piles, if the damping can be assumed to be concentrated at the pile tip, the actual damping resistance was shown in Equation 21 to be proportional to the pile properties (EA/c), bottom velocity (which can be calculated from the top velocity, pile properties and total driving resistance), and a damping constant J_c which is related to the soil type at the pile tip.

Data had been obtained on over 100 test piles (as of 1977) where static load test capacity, sufficient soil borings, and total driving resistance RT are available.

For each pile a damping constant J_c was calculated which produced a prediction which was equal to the static load test value. In addition, damping constants which produce only 20 percent error in Case Method prediction were determined. Any damping constant J_c chosen between these two limiting values will give a Case Method prediction which will be in error from the static load test value by less than 20 percent. Negative damping constants are physically meaningless and are therefore set to zero, should they occur. A plot of the non-negative damping constant J_c within 20 percent of the load test value is given as a function of soil type regardless of pile type in Figure 3. For piles with an ultimate capacity less than 150 kips, the acceptable error used was 30 kips. Measurement errors in the dynamic and static tests as well as the type and interpretation of the static load test failure (the Davisson failure criteria for rapid static testing was used as the failure definition) appear to dictate, for these low capacity piles, that an error range of 30 kips instead of 20 percent be considered acceptable. This additional range in the acceptable damping constant value is indicated in Figure 3 with the dashed lines. It can be seen that as the soil grains become finer the damping constant, J_c, must become larger. This result appeared logical.

For any given soil type on any job site where a static test is also run, a dynamic test on the static test pile will give the correct damping constant which can then be used on all remaining dynamically tested piles driven to the same soil stratum (see Figure 1 for sample J_c adjustment). For job sites with no static test to correlate with, the previous experiences shown in Figure 3 can then serve as a guide in choosing the proper damping constant which should yield a Case Method prediction within 20 percent of the static test result with a good degree of confidence. Recommended values are J_c equal to 0.1 for sand, 0.15 for silty sand, 0.2 for sand silt, 0.25 to 0.4 for silt, 0.4 to 0.6 for silty clays and clayey silt, and 0.6 to 1.0 for clay. It should be noted from Equation 21 that as the damping constant is increased, the resulting static prediction becomes more conservative. Thus, to assure that the Case Method is conservative, a higher damping constant than would normally be associated with the soil type need only be selected. It can be seen that the above recommended values are within 20 percent or are at least conversative in all but three cases. Two of these cases were for very low capacitypiles in silty clay which were not production piling but rather were driven especially for project personnel in the early stages of the project, when measurement techniques were not as advanced. The third pile was within 25 percent. In general the results of the Case Method using damping proportional to the pile cross section properties EA/c appear very realistic. A plot of the predicted versus measured capacities using the Case damping constants from soil borings and Case Method force and velocity measurements is shown in Figure 4 and uses the above recommended damping constants. Even better, essentially perfect agreement could have been obtained using Equation 21 to obtain the proper J_c for each specific pile site. Equation 21 can also be used with total resistance derived from CAPWAP analysis to obtain J_c for a site if a load test is unavailable and the soils are fine grained.

The Case Method capacity indicates the soil resistances at the time of testing. Setup or relaxation effects which commonly occur in soils are evaluated by testing at the end of driving and during restrike some time later. Care has been taken to only include cases where resistance changes affecting the damping constant empirical correlation have been minimized (i.e. for end of driving by running a static test as quickly as possible using a CRP or other short interval loading program; or by restrike testing after sufficient wait time to allow resistance changes to occur if correlating with lengthy load sequences). Restrike testing is always recommended on at least one pile per job site to assess the soil's strength changes with time and the long term service load of the pile.

6 Energy

The standard Case Method measurements of force and acceleration can be used to determine the energy transferred into the pile from the driving system. After integrating the acceleration to obtain velocity, the energy can then be computed from

$$E = \int F(t)v(t)dt \qquad 24$$

By comparing this measured value with the theoretical potential, manufacturer's rated or kinetic energy just before impact, an energy transfer ratio may be obtained. This ratio is the efficiency of the entire driving system after all losses (i.e. friction, in elastic collisions, pile cushions, capblocks and helmets, and gas compressions) and is not the actual hammer efficiency. It is unusual for the measured transfer ratio to exceed 70%. Values of 40 to 60% are normal. A measured transfer ratio below 30% usually indicates a hammer in need of maintenance.

Sometimes transferred energy can be calculated by force or velocity alone, making use of Equations 1 and 24. This approach is valid only if reflections from soil resistances or cross section changes are not yet present in the data. If reflections are present, Equation 24 is the only correct equation. This usually limits this technique to only very long, uniform section, off-shore piles with most of their length above the mudline.

7 Other Uses

The usefulness of measured force and velocity has been extended to other uses as well as capacity and energy determinations. For example, the measured forces can be inspected for their maximum compression and tension values. The force-velocity measurements can also be used to compute the maximum tension in the pile below the point of measurements from

$$T(x) = \frac{1}{2}(Iv(\frac{2L}{c}) - F(\frac{2L}{c}) - I_V(t_3) - F(t_3)) \qquad 25$$

where $t_3 = t_{max} + 2(L - x)/c$ and $I = EA/c$. Investigations of stresses are useful to prevent excessive pile damage or to improve driveability if stresses are too low.

Case Method measurements have often been used to inspect the structural integrity of the pile. Increases in velocity relative to the force at times before the computed 2L/c of the pile are the result of reduced cross sectional areas or stiffnesses, which for a uniform section pile is an unmistakeable indication of damage. The magnitude of this relative velocity increase can be related to the amount of damage.

APPENDIX A

Derivation of J_c

Replacing the pile by a mass-spring-damper system Newton's Second Law becomes

$$m\,\ddot{x}(t) + b\,\dot{x}(t) + k\,x(t) = F(t) \tag{A1}$$

If we now define (26)

$$\zeta = \frac{b}{2m\omega_n} \quad \text{and} \quad \omega_n^{\,2} = \frac{k}{m} \tag{A2}$$

where ζ is the viscous damping factor and b is the coefficient of viscous damping, then

$$\zeta = \frac{b}{\sqrt{mk}} \tag{A3}$$

where, for the pile, the mass m is ρAL and the stiffness k is $\frac{EA}{L}$. Recalling Equation 3.14, the value for ζ becomes

$$\zeta = \frac{bc}{2EA} \tag{A4}$$

and is critically damped when

$$b_{cr} = \frac{2EA}{c} \tag{A5}$$

introducing $J_c = 2\zeta$ we get

$$b = J_c\,\frac{EA}{c} \tag{A6}$$

for $0 < J_c < 2$.

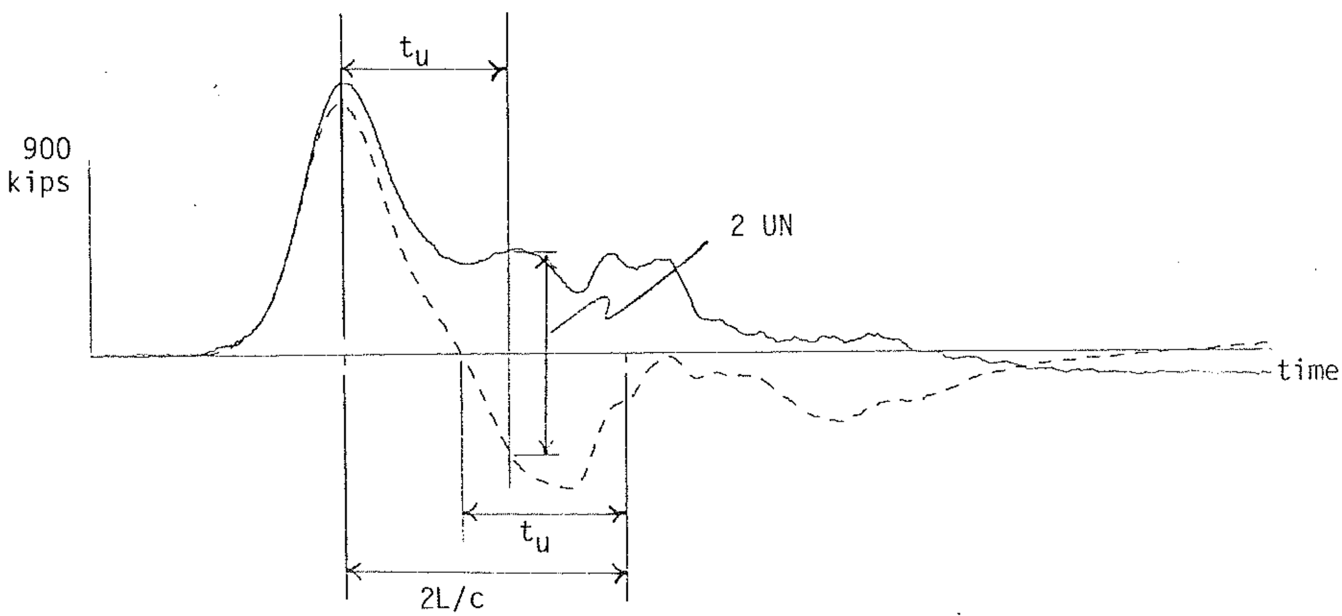
APPENDIX B

UNLOADING CORRECTION FOR THE CASE METHOD OF CAPACITY PREDICTION

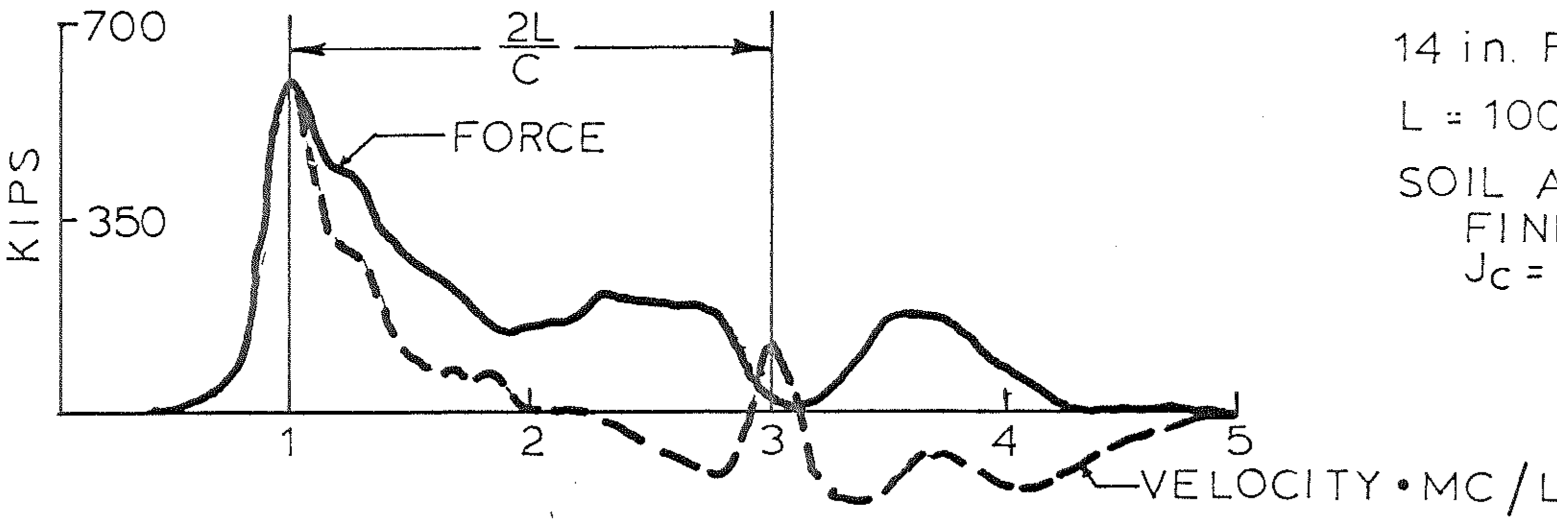
The Case Method of capacity prediction "measures" the resistance (capacity) acting simultaneously. For long piles having a significant portion of resistance coming from shaft friction, the Case Method may underpredict during hard driving, i.e. when the pile top rebounds. The pile top velocity becomes negative before the stress wave returns at time 2L/c. When this happens, the pile top is moving upwards and some skin resistance begins to unload.

The Case Method can be corrected in the following manner. (Refer to the accompanying figure.) First determine the difference between the time that the pile top velocity becomes zero and the stress wave return at 2L/c after impact. (Note that impact must be defined at the first velocity maxima.) This time, t_u, multiplied by the wavespeed c and divided by 2 represents the length of pile over which the unloading has occurred, l_u. To estimate the resistance that has unloaded, one measures the skin friction activated over the length, l_u. This is done on the force velocity plots by taking one half the difference between the force and velocity at time t_u after impact. In the example the unloading resistance, UN, is 468 kips. Adding UN to the RT (J=0) prediction of 767 kips gives a total driving resistance of 1235 kips. The dynamic component is then subtracted.

$$RS = RT + UN - J(2F1 - RT - UN)$$



Scales: Force 900 kips/inch
Velocity times EA/c 900 kips/inch
Time 12.5 msec/inch



14 in. P.S.C. (OCTAGONAL)

L = 100 ft

SOIL AT TIP
FINE SAND
$J_c = 0.1$

$$P4 = \frac{F1+F2}{2} + \frac{Mc}{2L}(V1 - V2)$$
$$= (601+17)/2 + (589-111)/2$$
$$= 309 + 239$$
$$= 548 \text{ KIPS}$$

$$D = J_c \bullet (2F - P4 - STAT)$$
$$= J_c \bullet (1202 - 548 - 12)$$
$$= J_c \bullet 642$$

$$R_u = P4 - D$$
$$= 548 - 0.1 \bullet (642)$$
$$= 484 \text{ KIPS}$$

$$STATIC = 470 \text{ KIPS}$$

$$R_u = P4 - D$$
$$470 = 548 - J_c(642)$$
$$J_c = 0.121$$

Figure 1

$$V(t) = \frac{-c}{EA} \sum_{i=1}^{n} R_i \left[2m + H\left(t - \frac{2x_i + 2mL}{c}\right)\right]$$

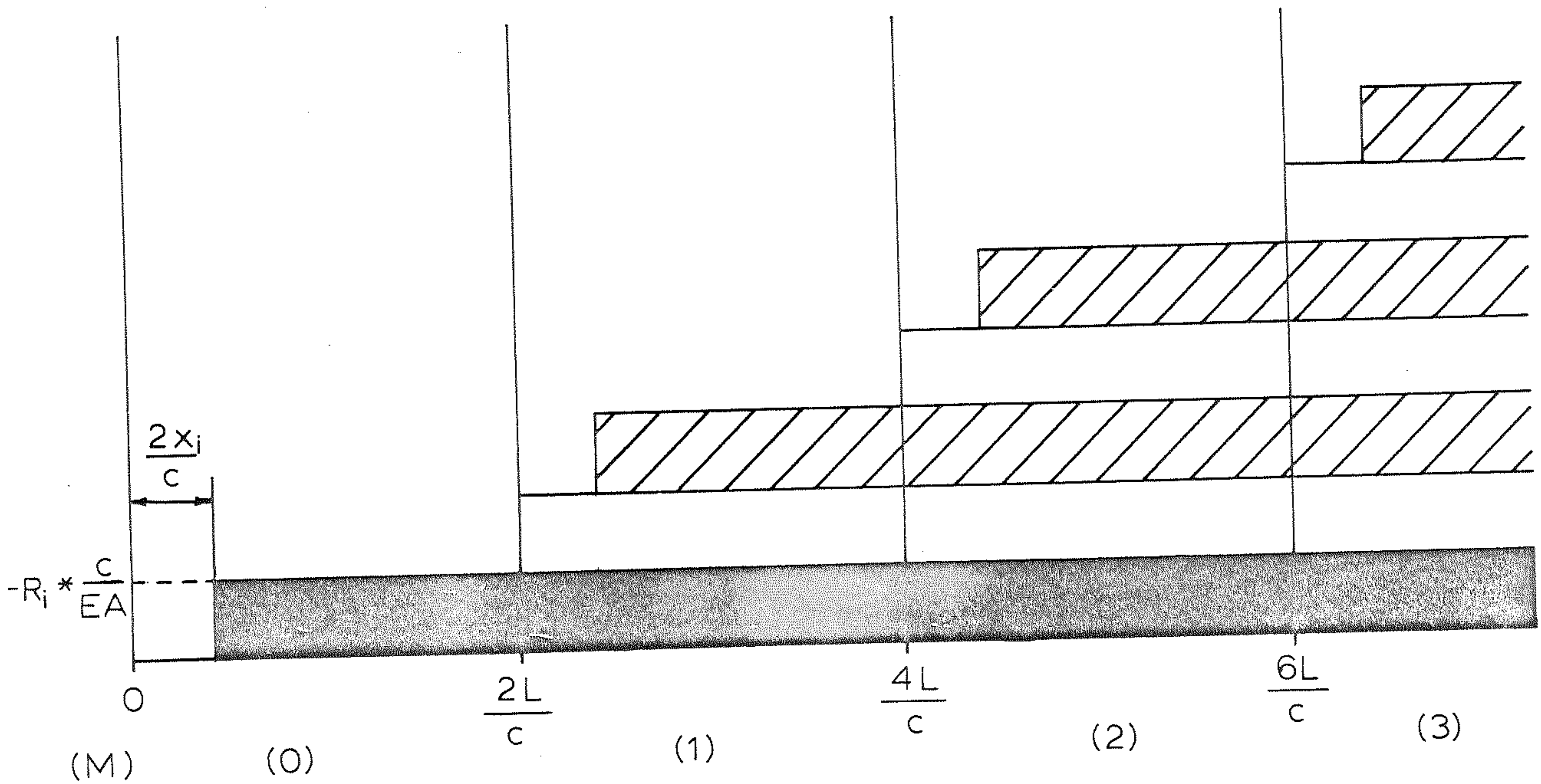


Figure 2: Velocity effect at pile top caused by a resistance step force R_i at location X_i from top.