# A SUMMARY OF THE PILE DRIVING ANALYZER CAPACITY METHODS - Past and Present -

By

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## Introduction

The Pile Driving Analyzer is a field computer that applies Case Method equations on measured pile dynamic data in order to determine, among other quantities (1), the pile's ultimate static bearing capacity. The Case Method was developed during the 1960s at Case Institute of Technology in a research project directed by Professor G. G. Goble. The research project went through several stages of development along two lines (a) improving the electronic equipment required for both data acquisition and numerical treatment; (b) refining the theoretical basis of capacity prediction equation. Today, the Pile Driving Analyzer with specially designed dynamic strain transducers and accelerometers, along with several closed form Case Method solutions provide on-site real-time predictions of pile capacity that has been proven to agree closely with those determined by static load tests (2). A brief background on the Case Project will be followed by the various Case Method procedures for computing ultimate static bearing capacity.

## History of Case Method Capacity

Beginning in 1964, research was conducted at Case Institute of Technology (now Case Western Reserve University) in Cleveland, Ohio to develop an economical, "practical for field use", easily portable system which bases

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pile bearing capacity predictions on electronic measurements of pile top force and acceleration during pile driving. Earlier research at Case (3) on model piles driven in sand considered the striking hammer and pile as a single rigid bodies to which Newton's Law was applied at the time of zero pile top velocity. Results gave a "strong reason for optimism" for the Case pile study. The most significant result of the small scale pile exercise was the research and testing of dynamic measurement equipment. With assistance from the Ohio Department of Transportation and the Federal Highways Administration, the project on the second stage acquired dynamic data on full scale piles that were also statically tested. This data bank contributed greatly to the success of the project.

Several phases were explored in analyzing that data to correctly predict static pile capacity as additional correlations with static load tests became available. The following reviews the different major phases.

Phase I Approach (P1)

$$R = F - Ma$$

Where F and a are functions of time and M is the total mass of the pile. In order to eliminate resistance force components dependent on pile velocity, F and a were chosen when the measured velocity was zero. Since results were sensitive to the exact time of zero velocity ($t_o$) time averaging was suggested in a second approach. The acceleration was integrated to velocity V; the difference in velocity at two times divided by the time period is then the average acceleration.

Phase II Approach (P2)

$$R = F(t_o) - M[v(t_o) - v(t_1)]/(t_o-t_1)$$

Where $t_1$ is the time of maximum pile top velocity and $t_o$ is the time of zero velocity. A solution which takes into account the travelling wave was developed. It was derived based on the assumption of a uniform pile cross section, linear elastic pile behavior, and one dimensional wave propagation. (7)

The times $t_o$ and $t_o + 2L/c$, where $2L/c$ is the time necessary for a wave to travel the pile length and return (L is the pile length, c is the stress wave speed), satisfied the wave theory and had minimum damping. The following equation represents this approach. (4)

Phase IIA (P2A)

$$R = 1/2[F(t_o) + F(t_o + 2L/c)] - M[v(t_o)+2L/c - v(t_o)]/(2L/c)$$

or

$$R = [F(t_o) + F(t_o + 2L/c) + \{v(t_o) - v(t_o + 2L/c)\} MC/L]/2$$

This approach worked well on piles driven into cohesive soils. All the above methods could be solved in analog electronics, making them ideal for the electronic analog computers used in the University project (i.e., the first Pile Driving Analyzers - Models A and B). The data after conversion to digital form could be subjected to more complicated analysis. Studies (5) with the "modified delta curve" led to the Phase III method.

Phase III (P3)

$$R = 2\, v(t)$$

where time t is chosen such that

$$2\, v(t) = [F(t) + F(t + 2\, L/c) + \{v(t) - v(t + 2L/c)\} Mc/L]/2$$

Unfortunately this can not be solved in an analog manner and this procedure was therefore not further considered.

For application in noncohesive soil, however, the Phase IIA method produced results which were too conservative. It was found empirically that the time of first relative maximum pile top velocity $t_{max}$ is a good time to begin the evaluation for noncohesive soils. Therefore, the following approach was adopted:

Phase IV approach (P4)

$$R = [(F(tmax) + (F(tmax + 2L/c)) + \{v(t_{max}) - v(t_{max}+2L/c)\}Mc/L]/2$$

where tmax was used for noncohesive soils (PI under 5) while $t_o$ was used for cohesive soils (PI over 10). For soils with PI between 5 and 10 a weighted average was given between the two results. Pile Driving Analyzer Models C and D used these methods. Different starting times instead of tmax for the computation (referred to as "time delay methods" were also used.

During the last two years of the research project at Case, the method was further modified to include a damping factor which was related to the soil at the toe. The data base primarily contained small diameter closed end pipe piles with a significant portion of their resistance at or near the pile toe (6).

The damping methods showed the best general correlation and was the most rational method proposed. Therefore, it became the standard computation (it could also be performed in analog form) and is still used today. All previous methods were abandoned.

The basic Case Method equation expressed as a function of pile top force and velocity is:

$$RT = [F(t_1) + F(t_1 + 2L/c) + \{V(t_1) - V(t_1 + 2L/c)\}Mc/L]/2$$

where RT is the sum of a static (RS) and a dynamic (D) component.

The "damping force" (D) can be obtained approximately as $D=J_c\ Mc/L\ V_{toe}$ where $J_c$ is a damping constant (Case Damping) and $V_{toe}$ is the pile toe velocity. Using the term Z for the impedance term Mc/L; it can be shown from wave propagation theory that the pile toe velocity can be calculated as:

$$V_{toe} = \{F_{top} + Zv_{top} - RT\}/Z$$

Where $V_{top}$ is the pile top velocity at time $t_{max}$. This equation is approximately correct for the first 2L/C time after the initial arrival of the stress wave at the pile toe. The static soil resistance (RS) is then obtained by subtracting the calculated damping force (D) from the total driving resistance (RT). (8)

$$RS = RT - D = RT - Jc(F_{top} + V_{top} - RT)$$

which can also be written in terms of force and velocity alone as:

$$RS = [(1 - J_c)(F(T_1) + Z\ V(t1)] + (1 + J_c)(F(t2) - Z\ V\ (t2))]/2$$

where $t_2 = t_1 + 2L/c$

The damping factor, $J_c$, was computed directly from the above equation when RS was substituted by the failure load as determined by a static load test according to the Davisson's failure criteria. The damping factor was found to be related to the soil grain size. The following values are suggested for different soil conditions:

0.10 to 0.15 for clean sands
0.15 to 0.25 for silty sands
0.25 to 0.40 for silts
0.40 to 0.70 for silty clays
0.70 to 1.00 for clays

The measured Force and Velocity may be used to determine the forces in the upward and downward travelling waves from:

$$WDN(t) = [F(t) + Z\ v\ (t)]/2$$
$$WUP(t) = [F(t) - Z\ v\ (t)]/2$$

The total resistance RT can also be computed from WDN and WUP at a time 2L/c later. The static component, RS, is:

$$RS = (1 - J_c)[WDN(t_1)) + (1 + J_c)[WUP(t_2)]$$

The Pile Driving Analyzer provides computations for the pile driving resistance and static bearing capacity with various methods of evaluating the measured pile force and velocity histories. The following will be discussed: Total Driving Resistance (RTL); Damping Factor Methods (RS1, RS2, RSM); Maximum Resistance Method (RMX); Minimum Resistance Method (RMN); Unloading Method (RSU), and a method for estimating total skin friction (SFT). Each procedure will be illustrated along with numerical example calculations. It should be emphasized that this article is not a manual for PDA users, but rather a guide line for the applicability of each method depending on the characteristics of the shape of the force and velocity measurements.

## I. Total Driving Resistance, RTL

The basic Case Method formula originally computed the total pile driving resistance to rapid penetration. This is now represented by the RTL procedure with computations assuming zero damping. The equation can be simply stated as a function of force and velocity as:

$$RTL = (F1 + F2 + V_1 - V_2)/2$$

or as a function of wave up and wave down as:

$$RTL = WDN1 + WUP2$$

where $F_1$ and F2 are force values at times 1 and 2, respectively. $V_1$=Z $V(t_1)$ and $V_2$=Z $V(t_2)$ and WDN1 and WUP2 are the values of the wave down at time 1 and wave up at time 2, respectively.

Time 1 is generally taken as the first relative maximum velocity peak and time 2 is 2L/c later. The location of time 1 along the record may be varied by either the "Delta" setting, or the "PEAK" selection switch on the Analyzer. Numerical examples for computing RTL from force and velocity, or wave traces are illustrated in Figure 1.

## II. Damping Factor Methods, RS1, RS2, RSM

All of these methods result in an estimate of the static bearing capacity given pile top force and velocity records. Basically, the equation can be expressed as:

$$RS = RTL - J_c \ (F_1 + V_1 - RTL)$$

where RS is the static bearing capacity and RTL is total driving resistance, $J_c$ is Case damping factor.

Depending on the selection of the "PEAK" switch, the RS values may be expressed as RS1, RS2, or RSM. RS1 is the standard Case Method which assumes time 1 to be at the first relative velocity peak. It is generally applied to traces that resemble those in Figure 2. The application of the above equation is shown in Figure 2.

The static capacity may also be expressed in terms of waves as:

$$RS = (1 - J_c)(WDN1) + (1 + J_c)(WUP_2)$$

An application of this formula taking time 1 to be the first relative velocity peak (i.e., RS1 method) is shown in Figure 3.

Both the total driving resistance RTL and the static capacity RS may be plotted as a function of time by applying their corresponding equations at different time increments as in Figure 4.

In some cases, it may be necessary to delay time 1 until either a second major velocity peak occurs. Particulary if force and velocity are proportional and are of similar or larger magnitude, choosing these peaks will generally give a better estimate of capacity. If the second peak is chosen, then the static capacity is expressed as RS2 and is illustrated in Figure 5. If a maximum peak is chosen, then the static capacity is RSM.

Caution should be used when using RSM since during easy driving, the maximum pile top velocity is often not at impact, but rather a reflection effect from the pile toe. The RSM method is illustrated in Figure 6.

## III. Maximum Resistance Method, RMX

In most cases, the velocity integral (i.e., displacement) at the first arrival of the peak input at any point along the pile is larger than the soil quake, assuring that the full resistance is mobilized. In "large quake" cases, it may be necessary to delay time 1. This delay allows the full capacity to be mobilized by waiting for extra displacement during the blow to be achieved. This method may also be appropriate in cases of small impacts or short rise times. The computations are done the same way as for the RS procedure with 2L/c time being fixed and varying the location of time 1 until the RS equation gives a maximum value, it is then called RMX. This procedure is illustrated in Figure 7. The time delay that was required, expressed as TMX, is measured from time 1. As shown in Figure 8, the RMX value may also be found by inspecting the resistance versus time plots. This RMX procedure is most suitable for evaluating the static bearing capacity of displacement piles driven into saturated fine grained soils. It might also be applicable for nondisplacement piles that are plugged. To judge if this method is applicable, the engineer must first recognize the existence of large soil quakes by observing the force and velocity records. The RMX should, however, be used with care in soils with high damping factors (greater than 0.4), or when the TMX value is more than 10 m sec.

Generally, large quakes are confirmed in records of high blow count where a 2L/c velocity increase is clearly observed.

## IV. Minimum Resistance Method, RMN

The speed of stress wave propagation in steel is constant at 16800 ft/sec. However, for concrete or timber, the wave speed varies from pile to pile, or even during the driving of a single pile. If the blow count is low (less than 40 blows per foot), the minimum resistance method (RMN) may be used with confidence.

The RMN equations are the same as those used for the RS methods except that time 2 is varied. The RMN computation uses the first or second peak, as selected by "PEAK" switch, as the time 1. The "Delta" dial has no effect for this method. Time 2 is varied through a $\pm$20% of 2L/c window that is centered around the selected 2L/c time. The RS equations are then searched always for their minimum value. The separation between times 1 and 2 is printed by the PDA as TMN which is effectively the 2L/c time used.

A numerical example for using the RMN method is given in Figure 9.

V. Unloading Method

The previous capacity methods all assume simultaneously acting soil resistance. For long flexible piles having a significant portion of resistance coming from distributed shaft friction, the Case Method standard equations may underpredict the pile capacity during hard driving if the pile top velocity becomes zero before time 2L/c. That means the pile top is moving upwards and some skin friction is beginning to unload even before the resistance effect near the pile toe has returned to the top.

In such cases, the PDA corrects the Case Method determined capacity in the manner shown in Figure 10. First, the time between zero velocity and 2L/c is determined, tu. The difference between force and velocity at a time tu after impact (2UN) represents twice the skin friction correction. The new estimate of the total static bearing capacity is computed by:

$$RSU = \{RTL + UN\} - J_c\ (F_1 + V_1 - \{RTL - UN\})$$

A numerical example that follows this procedure is shown in Figure 10.

VI. Total Skin Friction, SFT

The Pile Driving Analyzer uses the following procedure in estimating the total (dynamic and static) skin friction given pile top measurements of force and velocity histories during a hammer blow.

Time "a" is selected as 2L/c after the beginning of the rise in the force and velocity records (as in Figure 11). The rise time, tr, which is the time between initial rise and the peak, is then determined. This rise time is then used to determine point "b" which is one rise time earlier than time "a."

At Time "a" the separation between force and velocity (Fa - Va) represents the total pile skin friction above that point on the pile. The reason for moving point "a" one rise time before 2L/c is to exclude any reflections from the pile tip of the initial input wave. Then it is assumed that the skin friction for the bottom one rise length is the same as that one additional rise time length before. So the difference between the separation of forces and velocities at points a and b is added to the separation at point a. Simply stated:

$$SFT = (Fa - Va) + [(Fa - Va) - (Fb - Vb)]$$

This procedure along with a numerical example is illustrated in Figure 11.

Summary

Determination of pile driving resistance and static capacity is done by the Pile Driving Analyzer according to the Case Method equations. With the exception of the RTL and SFT procedures, all others discussed here, use a damping factor for static capacity computations. It is, therefore, very important that the measured pile force and velocity traces be evaluated according to the appropriate procedure with the applicable damping factor in order to accurately estimate the piles bearing capacity. It is worth noting that during very hard driving (in excess of 15 blows per inch) the PDA estimate of the pile capacity may be conservative since the full capacity may not be activated by this low displacement. Finally, as with any other method, the PDA capacity is that of the pile at the time of testing; time dependent soil strength changes (skin set up or toe relaxation) may only be assessed by testing the pile during restrike after an appropriate waiting period.

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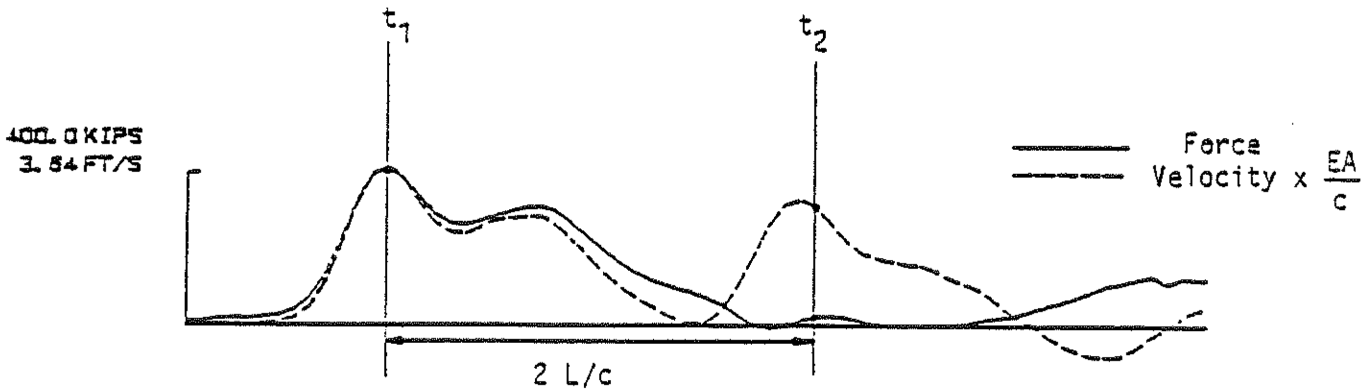



$t_2 = t_1 + 2\ L/c$

$F\ (t_1) = F1 = 415$ kips

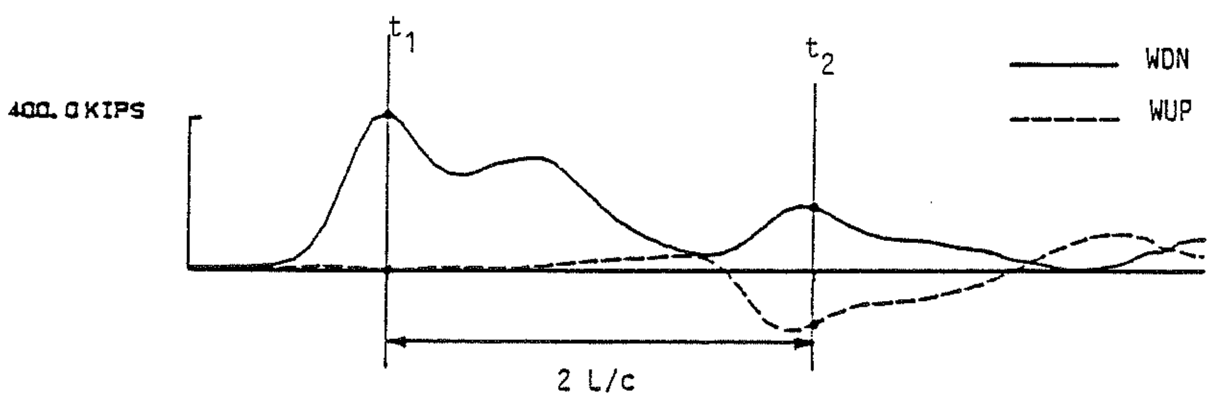
$F\ (t_2) = F_2 = 70$ kips

$V\ (t_1) \times EA/c = V1 = 415$ kips

$V\ (t_2) \times EA/c = V_2 = 350$ kips

$RTL = 1/2\ (F1 + F2 + V1 - V2)$

$= 1/2\ (415 + 70 + 415 - 350) = 275$ kips




$WDN = 1/2\ (F + V)$

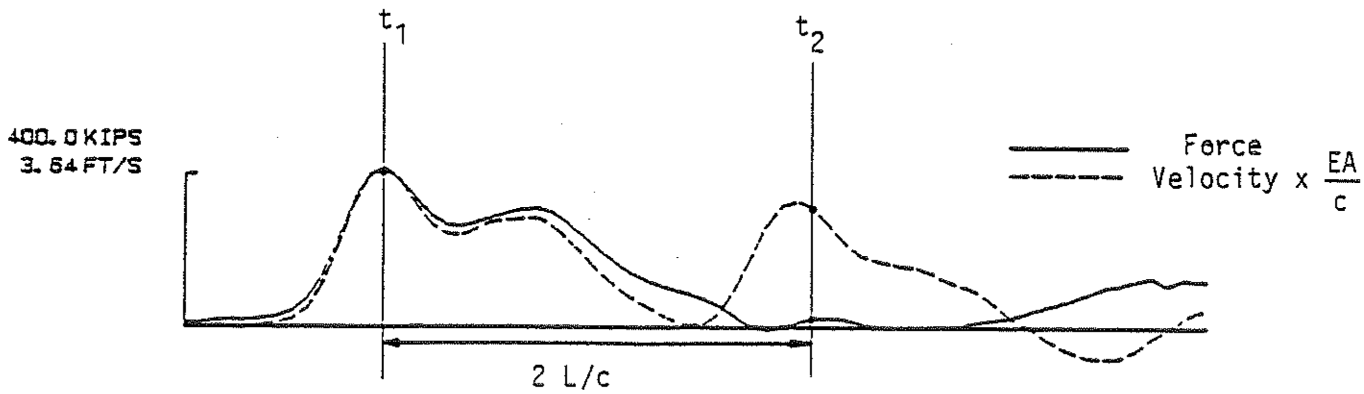
$WUP = 1/2\ (F - V)$

$WDN\ (t_1) = WDN1 = 415$ kips

$WUP\ (t_2) = WUP2 = -\ 140$ kips

$RTL = WDN1 + WUP2^*$

$= 415 - 140 = 275$ kips




$F\ (t_1) = F1 = 415$ kips

$F\ (t_2) = F2 = 70$ kips

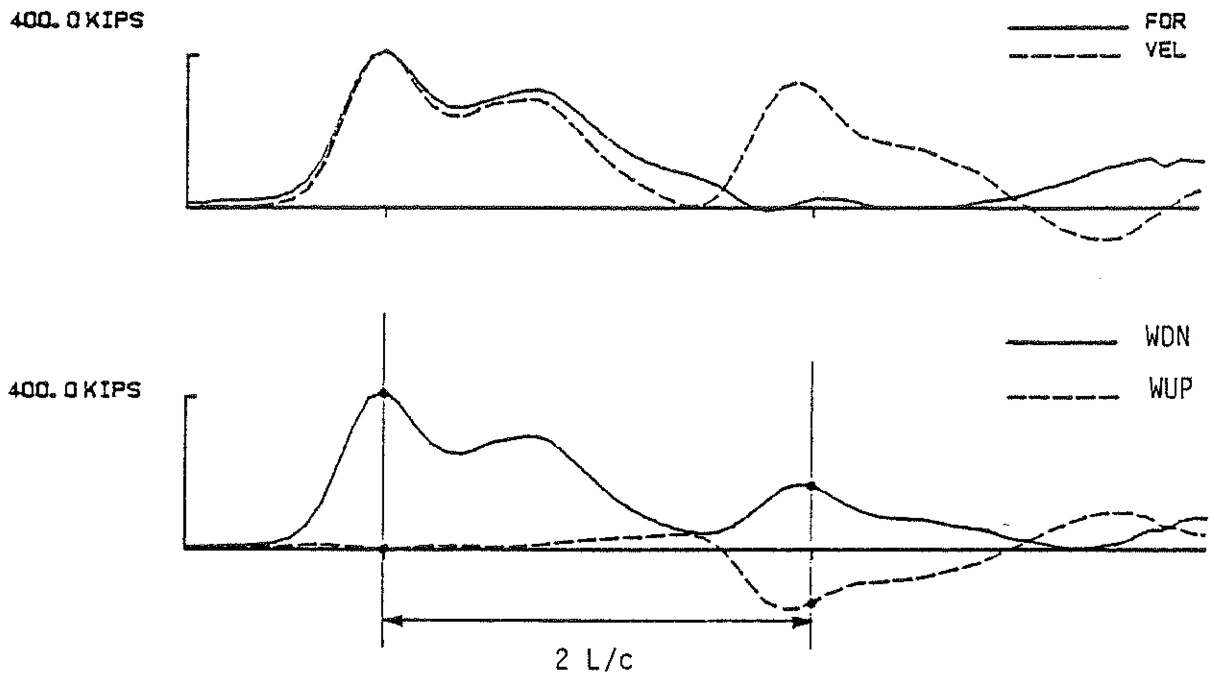
$V\ (t_1) = V1 = 415$ kips

$V\ (t_2) = V2 = 350$ kips

$$RTL = 1/2\ (F1 + F2 + V1 - V2)$$
$$= 1/2\ (415 + 70 + 415 - 380)$$
$$= 275 \text{ kips}$$

$$RS1 = RTL - J\ (V1 + F1 - RTL)$$
$$= 275 - 0.3\ (415 + 415 - 275)$$
$$= 108 \text{ kips}$$

RS1




WDN = Wave Down = 1/2 (F + v)

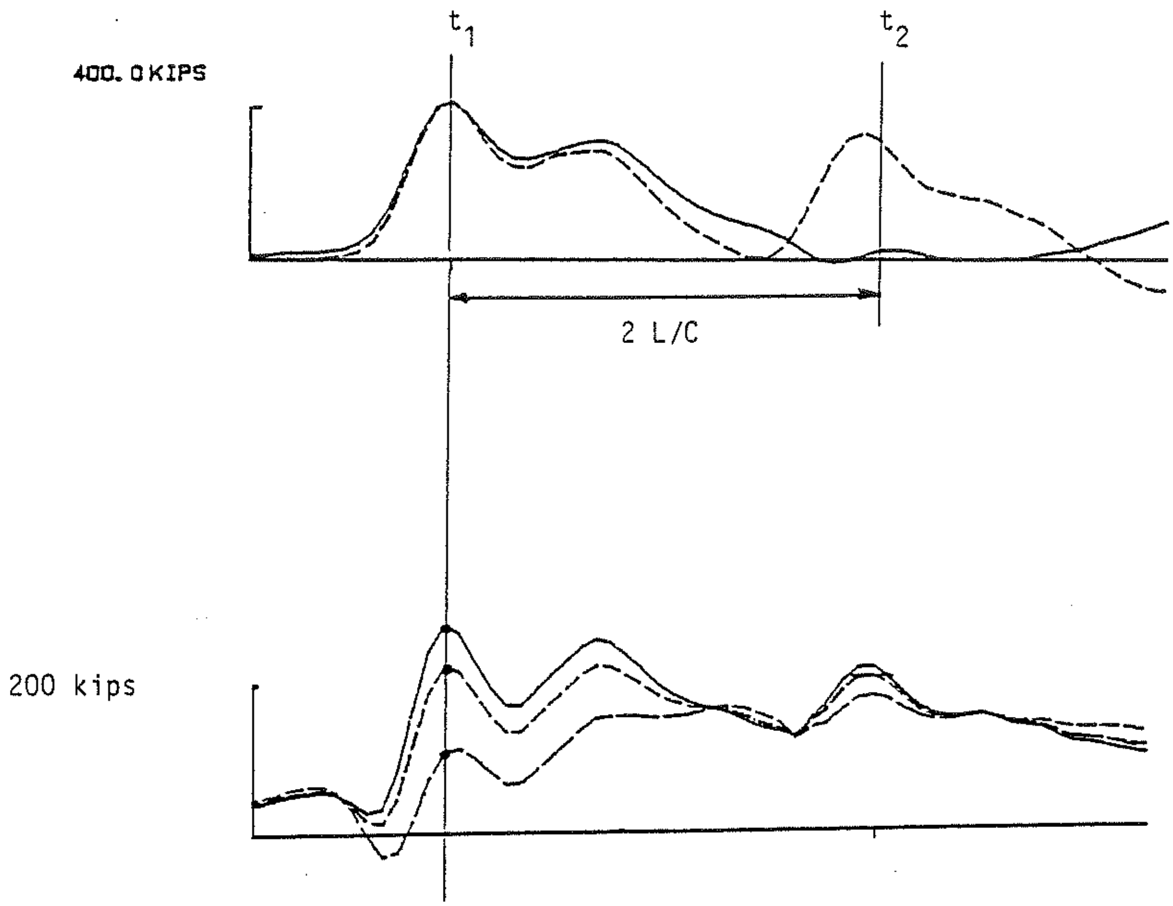
WUP = Wave Up = 1/2 (F-V)

WDN1 = 415 kips  WDN2 = 130 kips  WUP1 = 0  WUP2 =-140

RS1 = (1 - J) (WDN1) + ( 1 + J) (WUP2)

= (1 - 0.3) (415) + (1 + 0.3) (-140)

= 108 kips

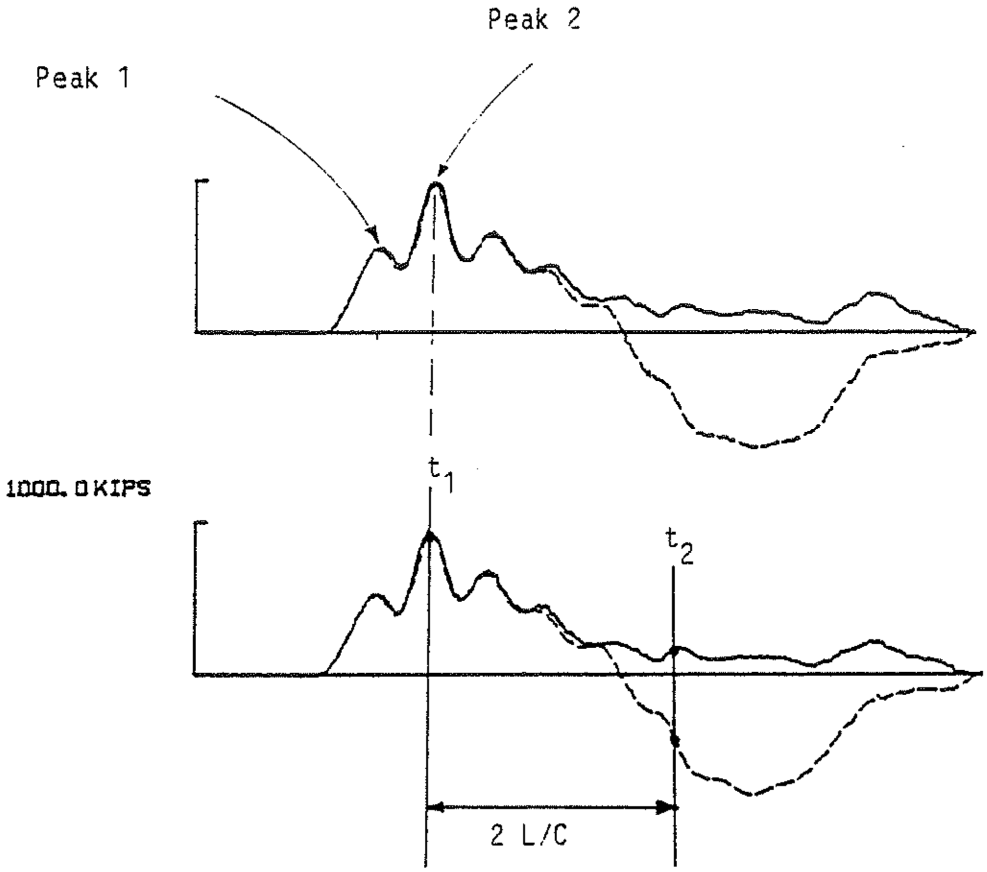



Total Resistance
Resistance with J = 0.1
Resistance with J = 0.3

From Resistance - time Curve:

RTL = 275 kips

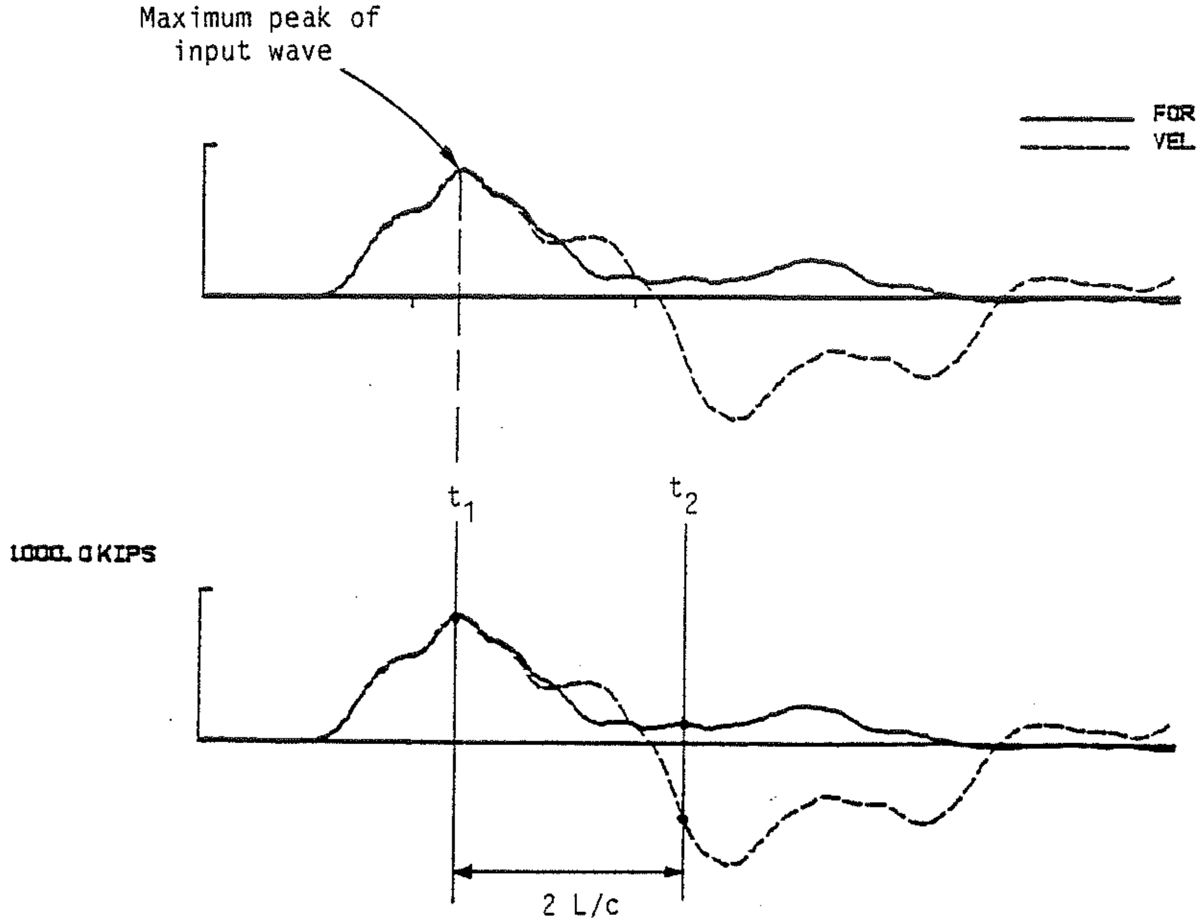
RS1, with J = 0.3, = 108 kips




F1 = 925 kips    F2 = 150 kips    V1 = 925 kips    V2 =-400 kips

$$RTL = 1/2\ (F1 + F2 + V1 - V2)$$
$$= 1/2\ (925 + 150 + 925 + 400)$$
$$= 1200 \text{ kips}$$

$$RS2 = RTL - J\ (V1 + F1 - RTL)$$
$$= 1200 - 0.3\ (925 + 925 - 1200)$$
$$= 1005 \text{ kips}$$




F1 = 800 kips    F2 = 130 kips    V1 = 800 kips    V2 = -500 kips

RTL = 1/2 (F1 + F2 + V1 - V2)

= 1/2 (800 + 130 + 800 + 500)

= 1115

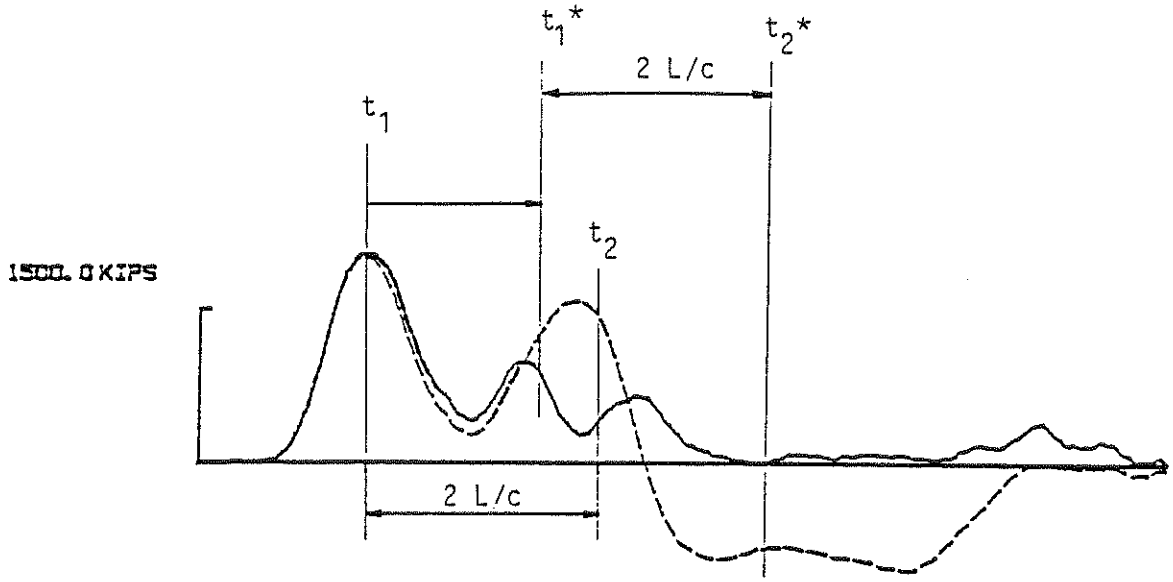
assume J = 0.2

RSM = RTL - J (F1 + V1 - RTL)

= 1115 - 0.2 (800 + 800 - 1115)

= 1018 kips

# RMX




F1 = 2045 kips F2 = 405 V1 = 2045 kips V2 = 1427

F1* = 878 kips F2* = 0 kips V1* = 1244 kips V2* = 842 kips

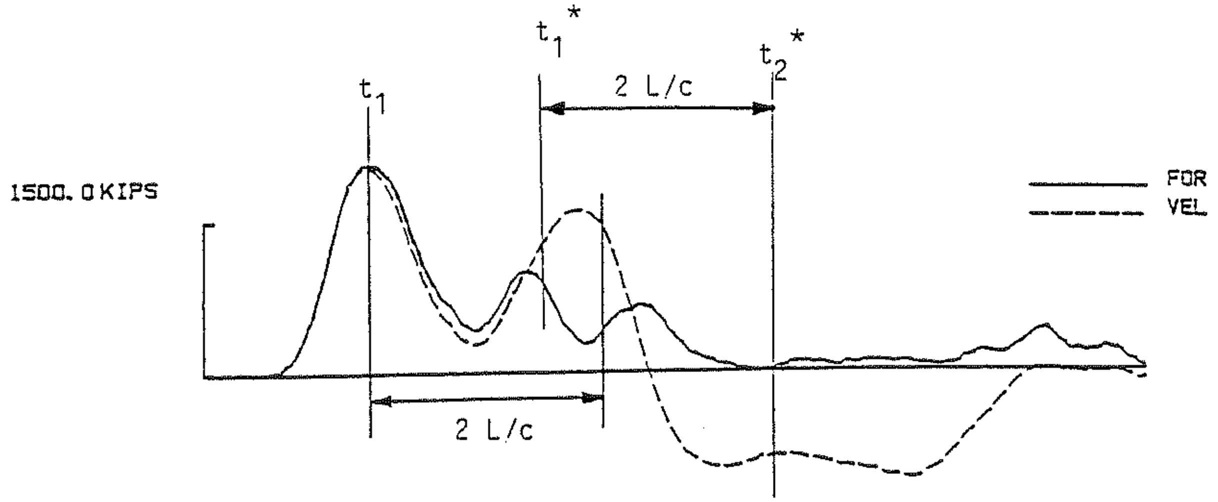
$$RTL^* = 1/2\ (F1^* + F2^* + V1^* - V2^*)$$
$$= 1/2\ (878 + 0 + 1244 + 842)$$
$$= 1482 \text{ kips}$$

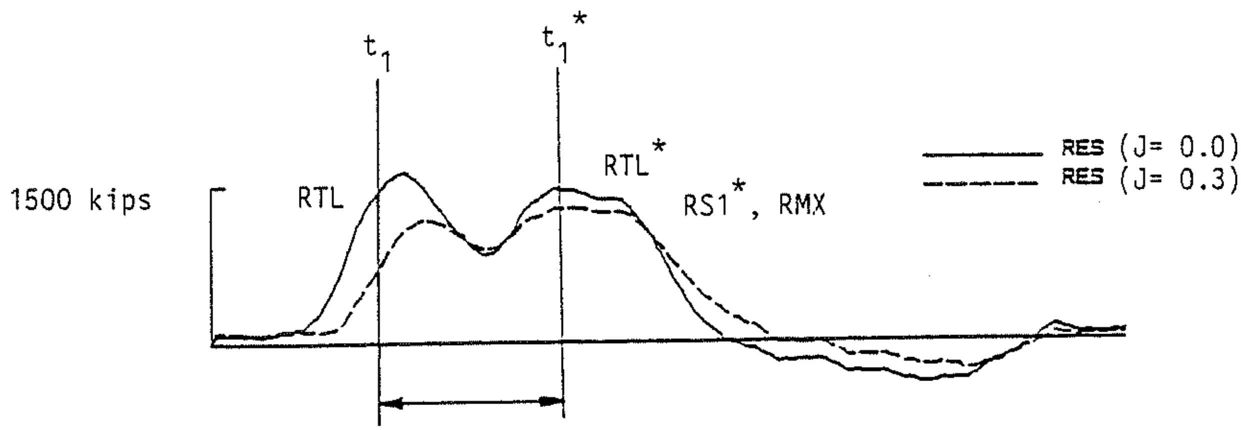
$$RS1^* = RMX = RTL^* - J\ (V1^* + F1^* - RTL^*)$$
$$= 1482 - 0.3\ (1244 + 878 - 1482)$$
$$= 1290 \text{ kips}$$

<u>For Comparison:</u>

$$RTL = 1/2\ (F1 + F2 + V1 - V2)$$
$$= 1/2\ (2045 + 405 + 2045 - 1427)$$
$$= 1534$$

$$RS1 = RTL - J\ (V1 + F1 - RTL)$$
$$= 1534 - .3\ (2045 + 2045 - 1534)$$
$$= 767 \text{ kips}$$




Reading directly from the Resistance - time plots:

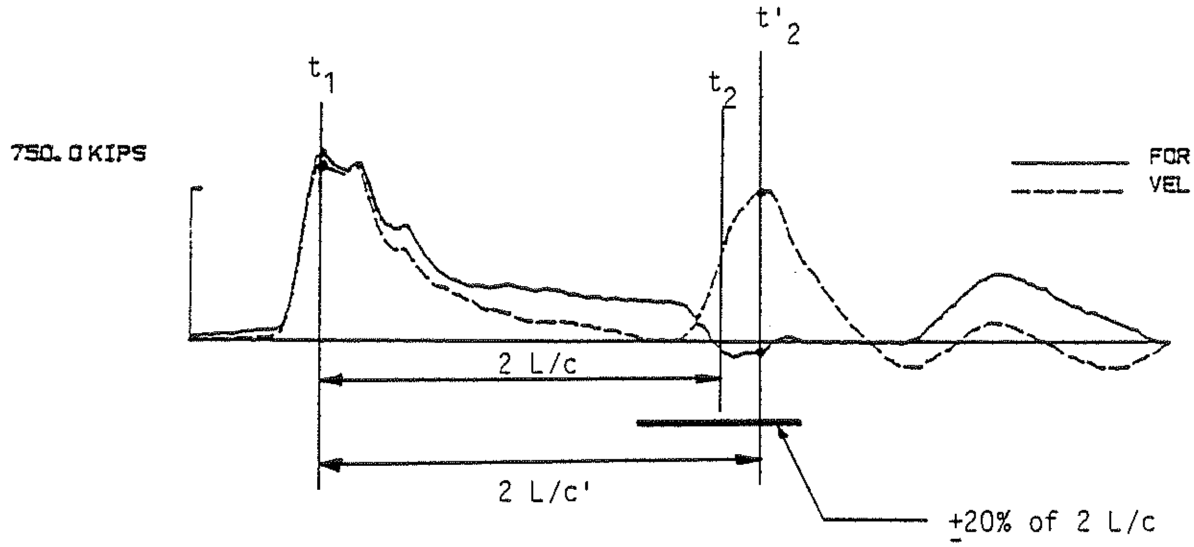
RTL = 1534 kips

RS1 = 767 kips

RTL$^*$ = 1482 kips

RS1$^*$ = RMX = 1290 kips

# RMN

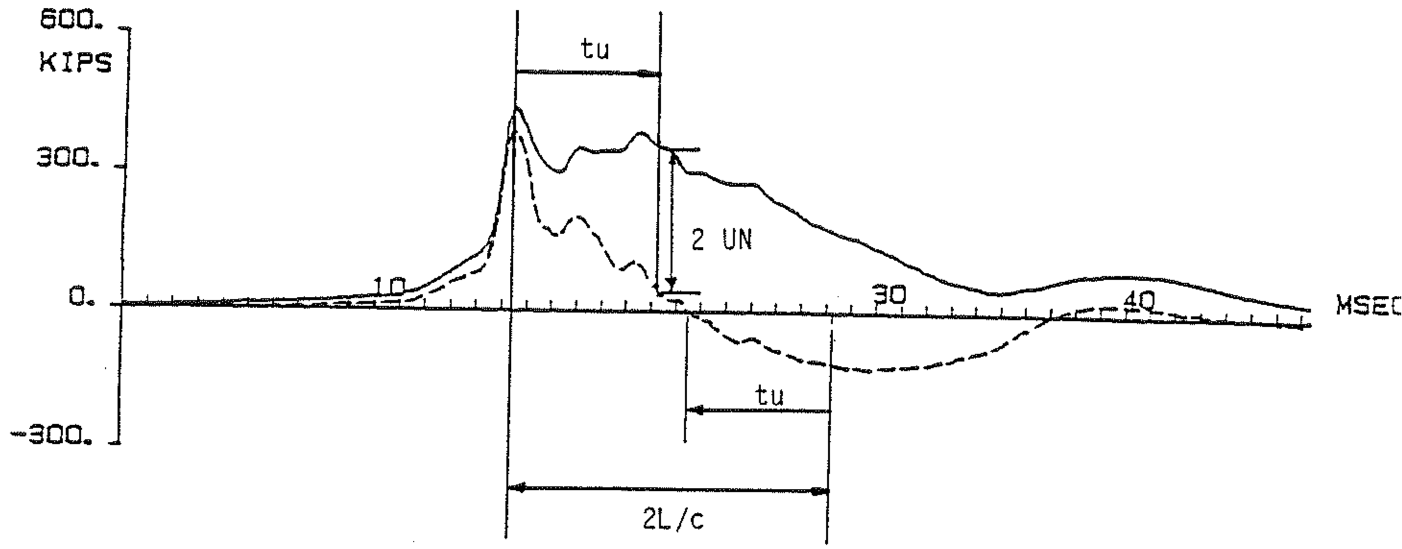



F1 = 937 kips     V1 = 892 kips     F2' = -60 kips     V2' = 750 kips

$$RTL' = 1/2\ (F1 + F2 + V1' - V2')$$
$$= 1/2\ (937 - 60 + 892 - 750)$$
$$= 509.5 \text{ kips}$$

assume J = 0.2

$$RS1' = RMN = RTL' - J\ (F1 + V1 - RTL')$$
$$= 509.5 - 0.2\ (937 + 892 - 509.5)$$
$$= 246 \text{ kips}$$




F1 = 450 kips F2 = 183 kips V1 = 400 kips V2 = 117 kips 2UN = 317 kips

RTL = 1/2 (F1 + F2 + V1 - V2)

= 1/2 (450 + 183 + 400 + 117)

= 575 kips

RSU = RTL + UN - J (2F1 - RTL - UN)

= 575 + 158.5 - 0.5 (2 x 450 - 575 - 158.5)

= 650.25 kips

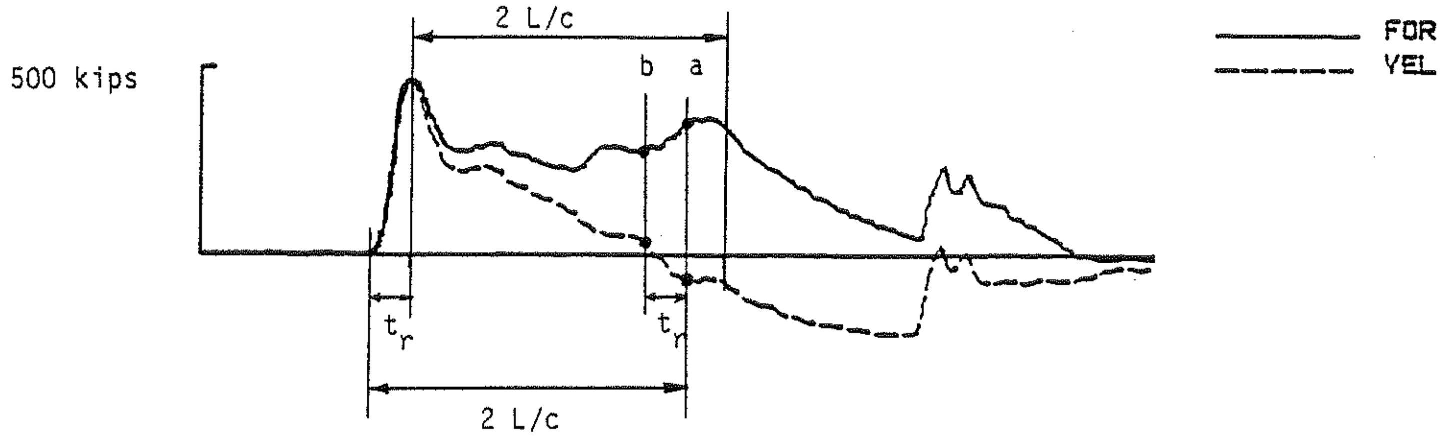
<u>For Comparison:</u>

RS1 = RTL - J (V1 + F1 - RTL)

= 575 - 0.5 (400 + 450 - 575)

= 437.5 kips

# SFT




$t_r$ is the rise time of the input wave

$F_a$ = 340 kips

$V_a$ = -60 kips

$F_b$ = 280 kips

$V_b$ = 40 kips

$$SFT = (F_a - V_a) + [ (F_a - V_a) - (F_b - V_b)]$$

$$= (340 + 60) + [ ( 340 + 60) - (280 - 40)]$$

$$= 560 \text{ kips}$$

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